Test 2

MAT 310/Zandieh
April 6, 1999
30 points

Calculators allowed.
No notes allowed.

DO NOT WRITE ANSWERS ON THIS PAGE
Write your answers on the blank pages provided.

1. (3 points) Consider a small triangle on a sphere. Let A_1, A_2, and A_3 be the area of the three lunes which share an angle with the triangle. Write an equation that relates the area of the triangle, A_T, the area of the three lunes, A_1, A_2, and A_3, and the area of the entire sphere, A_S. Explain why this equation accurately relates the given quantities. (Stop there. Do not do any additional algebra.)

2. (3 points) Explain what is meant by the holonomy of a triangle?
   List the angle measures of one triangle on a sphere with holonomy $\pi/3$.

3. (3 points) List all the symmetries of a lune on the sphere. If you list a reflection symmetry, state the line of reflection. If you list a rotation symmetry, state the point it is rotated about and the degree of the rotation. If there are reflection symmetries about more than one line or rotation symmetries about more than one point, state each of these. Are there any non-rotation or non-reflection symmetries?

4. (4 points) **State** and **prove** either *one* of the following theorems (SAS, ASA) on the plane.

   (3 points) On the sphere this theorem is only true for "small triangles". Why must we restrict ourselves to small triangles (remind me of your definition of small triangles)? Also, why needn't we restrict this theorem to "very small triangles" (i.e. how do you know it's true for all small triangles including those that are not very small)?

5. (2 points) Suppose x and x' are parallel transports along line m. Draw a picture that represents this situation (make sure to label x, x' and m) and state the definition of parallel transport.

   (3 points) On a sphere x and x' form a lune. Consider the distance from the vertex of this lune to the intersection of m and x and to the intersection of m and x'. Could both of these lengths (for the one fixed line m) be shorter than one-fourth of a great circle? Why or why not? Prove your assertion.

6. (4 points) **State** and **prove** either *one* of the following theorems (SAA, EEAT) on the plane.

7. (3 points) Consider your planar proof for SAA or EEAT in problem 6. This proof will work on the sphere if we restrict our triangles to very small triangles. Explain which part of the proof is not necessarily true for small and regular triangles on a sphere, but will work if the side lengths of triangles are restricted to less than one-fourth of a great circle. Make sure to

include the reason why this restriction is necessary. (Your answer to the above two problems may help you answer this.)

8. (2 free points) What is your intended occupation after graduating from ASU?