

**Zeros of a Polynomial**

Algebraically solve  $f(x) = 0$  for the following equations:

1.  $f(x) = x^3 - 7x^2 + 11x$ , given that 0 is a zero of  $f(x)$ .
2.  $f(x) = 2x^3 + 3x^2 - 89x + 120$ , given that 5 is a zero of  $f(x)$ .
3.  $f(x) = x^3 - 3x^2 - x + 3$ , given that 1 is a zero of  $f(x)$ .

**Zeros and Multiplicities**

Algebraically find all the zeros and their multiplicities for the following functions:

4.  $f(x) = 2x^3 - 3x^2 - 12x + 20$
5.  $f(x) = x^3 - 3x + 2$
6.  $f(x) = 3x^3 + 22x^2 + 15x - 100$

**End Behavior of Polynomials**

Use the leading coefficient test to determine the end behavior of the following polynomial functions:

7.  $f(x) = -2x^3 + x - 2$
8.  $f(x) = -x^{10} - 3x^9 + x^2$
9.  $f(x) = 2x^3 + x - 2$

**Long or Synthetic Division**

Divide the following using long or synthetic division:

10.  $\frac{2x^4 - 6x^2 + 1}{x + 1}$
11.  $\frac{4x^2 - 8x + 1}{2x - 1}$
12.  $\frac{2x^3 - 7x^2 + 2x + 3}{x - 3}$

**Vertical Asymptotes**

Find the equation of the vertical asymptotes (if any) of the following functions:

13.  $f(x) = \frac{3x + 2}{x^2 - 1}$
14.  $f(x) = \frac{x + 4}{3x + 1}$
15.  $f(x) = \frac{x + 2}{x^2 - 4}$

### Applications of Rational Functions

16. The following rational function in hundreds models the population of a certain species of animal, where  $t$  is measured in days. What number does the population approach in the long run?

$$p(t) = \frac{10t^3 + 2}{2t^3 + 1}$$

17. The average cost of producing a popular board game is given by the function:

$\bar{C}(x) = \frac{1500 + 15x}{x}$ ,  $x \geq 0$ , when  $x$  is the number of the board game sold, identify the horizontal asymptote of the function and explain its meaning in this context.

18. The function  $N(t) = \frac{0.8t + 100}{5t + 4}$ ,  $t \geq 15$ , gives the body concentration  $N(t)$ , in parts per million of a certain dosage of medication after time  $t$ , in hours. Find the horizontal asymptote of the graph and explain the meaning in the context of the problem.

### Rewrite in the equivalent logarithmic form

19.  $a^{x+1} = 65$

20.  $e^{3x} = 5$

### Rewrite in the equivalent exponential form

21.  $\log_6(4x) = 10$

22.  $\ln(B) = A$

### Compound interest

23. Find the accumulated value of an investment of \$21,000 at an interest rate of 5.6% for 7 years:

a) compounded monthly

b) compounded continuously

24. a) What initial investment at 3.75% interest compounded continuously for 10 years will accumulate to \$20,000? Round your answer to the nearest cent.

b) What initial investment at 4.25% interest compounded monthly for seven years will accumulate to \$20,000? Round your answer to the nearest cent.

### Properties of Logarithms

Use properties of logarithms to write as a sum or difference logarithms with no exponents.

25.  $\log\left(\frac{x^5 y^7}{z^3}\right)$

26.  $\ln\left((x-1)^{\frac{3}{2}} \sqrt{\frac{(y+3)^4}{z^8}}\right)$

Use properties of logarithms to express the following as a single logarithms

27. a)  $2\ln(x) - 5\ln(y) + 9\ln(w)$                       b)  $\frac{3}{2}\ln(x+3) - \ln(x) - \frac{1}{2}\ln(x+3)$

28. a)  $3\log(A) - 4\log(B) + 5\log(C) - 6\log(D)$                       b)  $\log(8) + \log(x^2 - 1) - \log(x) - \log(x+1)$

### Exponential Equations

Solve the following for  $x$

29.    a)  $2^{2x+17} = 8$                                       b)  $10e^{3x-7} = 5$

30.    a)  $e^{2x} + 2e^x - 35 = 0$                       b)  $(7)^{2x} + 2(7)^x - 15 = 0$                       c)  $2e^{2x} + 3e^x - 20 = 0$

### Domain of Logarithms function:

Find the domain of the following function;

31. a)  $f(x) = \ln(6 - 2x)$

      b)  $f(x) = \log(4x + 16)$

### Logarithms Equations

32. Find the x-intercept of the following function:

      a)  $f(x) = 4 - 2\log_3(2x - 10)$

      b)  $f(x) = \ln(2x + 3)$

33. Solve the following for  $x$ :

      a)  $\log_6(x) - \log_6(x - 5) = 2$

      b)  $\ln(x) + \ln(2x + 1) = 0$

### Applications of exponential equations:

34. How long will it take any quantity of iodine 131 to decay to 25% of its initial amount, knowing that it decays according to the function

$A(t) = A_0e^{-0.87t}$ , where  $t$  is the number of days?

35. The population of Merchantville was 20,000 in 1990 and 25,000 in 1995. If exponential growth is assumed, find a model for the population growth, and then use the model to determine the population in 2008.

36. A sample of 500 grams of radioactive lead 210 decays to polonium 210 according to the function  $A(t) = 500e^{-0.32t}$ , where  $t$  is in years. Find the amount of the sample remaining after

- (i) 4 years,                      (ii) 8 years.                      (iii) Find the half-life.

### More with polynomials and zeros

37. Identify the zeros and the multiplicities of each zero for  $f(x) = -2x^4(x+3)^2(x-7)^8$ .
38. Construct a degree 4 polynomial with real coefficients with zeros at  $3i$  (multiplicity 1),  $-4$  (multiplicity 2) and with leading coefficient of 1.
39. Construct a degree 3 polynomial with real coefficients, with zeros at  $2 + 3i$  (multiplicity 1),  $5$  (multiplicity 1), and with leading coefficient of 1.

### More with rational functions

40. Construct a rational function with the following characteristics:
- $x$ -intercepts at  $(2,0)$  and  $(7,0)$
  - vertical asymptotes at  $x = 4$  and  $x = -5$
  - horizontal asymptote at  $y = 9$

**Answers** (some of which have the possibility of not being incorrect)

1.  $0, \frac{7}{2} \pm \frac{1}{2}\sqrt{5}$     2.  $5, -8, \frac{3}{2}$     3.  $1, -1, 3$

4. zero at  $\frac{-5}{2}$  mult. 1 and zero at 2 mult. 2

5. zero at  $-2$  with mult. 1 and zero at 1 with mult. 2

6. zeros at  $-4, \frac{5}{3}, -5$  with mult. of 1 for each.

7. rises left, falls right    8. falls left, falls right    9. falls left, rises right

10. Q:  $2x^3 - 2x^2 - 4x + 4$     R:  $-3$     11. Q:  $2x - 3$     R:  $-2$

12. Q:  $2x^2 - x - 1$     R:  $0$

13.  $x = \pm 1$     14.  $x = -\frac{1}{3}$     15.  $x = 2$

16. 5 hundred

17. 15 is the H.A., the average cost when producing a great number of games is \$15.

18. 0.16 ppm is the concentration after a long time.

19.  $\log_a(65) = x + 1$     20.  $\ln(5) = 3x$     21.  $4x = 6^{10}$     22.  $B = e^A$

23. a) \$31,050.37    b) \$31,078.69    24. a) \$13,745.79    b) \$14,861.26

25.  $5\log(x) + 7\log(y) - 3\log(z)$       26.  $\frac{3}{2}\ln(x-1) + 2\ln(y+3) - 4\ln(z)$

27. a)  $\ln\left(\frac{x^2w^9}{y^5}\right)$     b)  $\ln\left(\frac{x+3}{x}\right)$       28. a)  $\log\left(\frac{A^3C^5}{B^4D^6}\right)$     b)  $\log\left(\frac{8x-8}{x}\right)$

29. a)  $-7$     b)  $\frac{7 + \ln(1/2)}{3} = \frac{7 - \ln(2)}{3} \approx 2.102$

30. a)  $\ln 5 \approx 1.609$     b)  $\frac{\ln(3)}{\ln(7)} = \log_7(3) \approx 0.56458$     c)  $\ln\left(\frac{5}{2}\right) \approx 0.916$

31. a)  $(-\infty, 3)$     b)  $(-4, \infty)$     32. a)  $(\frac{19}{2}, 0)$     b)  $(-1, 0)$     33. a)  $\frac{36}{7}$     b)  $\frac{1}{2}$

34. 15.93 days    35.  $P = 20000e^{0.044629t}; 44,658$

36. i) 439.93 gm    ii) 387.07 gm    iii)  $\frac{\ln(2)}{k} = \frac{\ln(2)}{0.032} = 21.66$  years

37. 0, multiplicity 4; -3, multiplicity 2; 7, multiplicity 8

38.  $p(x) = (x^2 + 9)(x + 4)^2$       39.  $p(x) = (x - 5)(x^2 - 4x + 13)$

40.  $r(x) = \frac{9(x-2)(x-7)}{(x-4)(x+5)}$