Math 598, Spring 2006

HOMEWORK 2

(due Thursday, February 23)

1. Prove the Young's inequality
(*Remember to write out this problem in a proper math manner!*):
Given $f \in L^p(\mathbb{R}^n)$ and $g \in L^q(\mathbb{R}^n)$, show that

$$\|f * g\|_{L^r} \leq c\,\|f\|_{L^p}\|g\|_{L^q},$$

provided $\frac{1}{p} + \frac{1}{q} = 1 + \frac{1}{r}$ and $1 \leq p, q, r \leq \infty$.

2. Compute the convolution $f * g(x)$, where

(a) $f(x) = e^x$ and $g(x) = \chi_{[0,\infty)}(x)$

(b) $f(x) = e^{-|x|}$ and $g(x) = \sin x$

(c) $f(x) = x \cdot \chi_{[-2,1]}(x)$ and $g(x) = x^2 + 2$

(d) $f(x) = \chi_{[-4,0]}(x)$ and $g(x) = \chi_{[-1,1]}(x)$

3. Compute $f * f$ and $f * f * f$ for $f(x) = \chi_{[-1,1]}(x)$.

4. Let f and g be step functions. Show:

(a) $\mathrm{supp}\,(f + g) \subseteq \mathrm{supp}\, f \cup \mathrm{supp}\, g$

(b) $\mathrm{supp}\,(f \cdot g) \subseteq \mathrm{supp}\, f \cap \mathrm{supp}\, g$

(c) $\mathrm{supp}\,(\lambda \cdot g) \subseteq \mathrm{supp}\, f$, $\lambda \in \mathbb{R} \setminus \{0\}$

(d) What can you say about $\mathrm{supp}\,(f * g)$?