Practice Final

The final exam will cover ALL the material covered this semester. This exam is just a SAMPLE and NOT ALL of the material is covered by the problems in this practice test. Review ALL sections:

- Chapter 2 (2.1-2.3, 2.5, 2.6)
- Chapter 3 (all sections)
- Chapter 5 (5.1, 5.2, 5.3)
- Chapter 6 (6.1, 6.2)
- Chapter 7 (7.1-7.3, 7.5, 7.6 and 7.8)

Practice Problems

1. Find the solution to the initial value problem $y' = e^x y^2 + 2xy^2$, $y(0) = 1$. Your answer should be in explicit form: $y = f(x)$.

2. Find the general solution to $\dfrac{dy}{dx} = \dfrac{-3xy^2 - \cos x}{3x^2 y + \sin y}$. You may leave your answer in implicit form.

3. A spring is stretched a distance of $\dfrac{32}{9}$ *ft* by a mass m which has weight $mg = 2$ *lbs*, where $g = 32$ *ft/sec*^{2} is the acceleration due to gravity. The spring is at rest until time $t = 0$, when an external force $F(t) = 3\sin 3t$ *lbs* is applied to the spring. Assuming no damping, find the position $u(t)$ of the spring at time t, where $u = 0$ corresponds to the rest position of the spring.

4. Find the first 3 non-zero terms in the power series expansion at the origin for each of the two linearly indendent solutions of
$$y'' + 2xy' - 3y = 0.$$

5. Find all singular and ordinary points of the equation
$$(x-1)^2(x-3)^2 y'' + 8(x-1)y' + (x-3)y = 0.$$

6. (a) Find a first order linear system of differential equations which is equivalent to the second order equation $y'' + 2y' - 15y = 0$. If this system is written in the form $\mathbf{x}' = A\mathbf{x}$, find the matrix A.

   (b) Solve the system found in part (a). Use this to find the general solution y of $y'' + 2y' - 15y = 0$.

   (c) Find the general solution y of the equation $y'' + 2y' - 15y = 0$ and compare with the answer found in part (b).

   (d) Let W_1 be the Wronskian of the solutions to the system in part (a). Let W_2 be the Wronskian of two linearly independent solutions to $y'' + 2y' - 15y = 0$. Compute W_1 and W_2 and compare them.

7. (a) Find 2 linearly independent **real-valued** solutions to the system

$$\mathbf{x}' = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix} \mathbf{x}\,.$$

   (b) Select one of the solutions you obtained in part (a) and check that it in fact satisfies the system $\mathbf{x}' = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix} \mathbf{x}$.

   (c) Sketch the graph of solution trajectories.

8. Find the solution of the initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 5 & 1 \\ 3 & 5 \end{pmatrix} \mathbf{x}\,, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

9. Find the general solution of the system

$$\mathbf{x}' = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{pmatrix} \mathbf{x}\,.$$

   Sketch the graph of the general solution trajectories.

10. Consider the differential equation

$$(\star) \qquad (x^2 - 1)y'' - 6y = 0.$$

   (a) Find the recurrence relation for the coefficients in the series expansion of y at the origin.

   (b) Find the solution y of $(\star)$ that satisfies $y(0) = 0$ and $y'(0) = 1$.

   (c) Find a lower bound for the radius of convergence of the series around the origin for any solution of $(\star)$. Give reasons for your answer.

11. Find the first 3 non-zero terms in the series solution at $x_0 = 0$ for each of two linearly independent solutions of the differential equation

$$y'' - 2xy' - 2y = 0.$$

12. Give the definition and an example of: (a) Laplace transform, (b) steady-state solution, (c) ordinary point, (d) forced responce, (e) unstable node, (f) eigenvector.

13. Review all homework problems, quizzes, exam #1 and exam #2 (and their solutions which are posted on the web) and class notes.

Check the class website for the time and day of the final exam!! The exam will be in PSA 304.