Math 274 Info on Exam 2

The Exam 2 will cover:
- Chapter 3 (sections 3.8 and 3.9)
- Chapter 7 (sections 7.1-7.3, 7.5, 7.6, 7.8)
- Chapter 6 (sections 6.1, 6.2).
Table with the Laplace transform (as on p.319) will be included.

Practice Exam 2

1. (a) A mass weighing 1 *lb* stretches a spring $\frac{1}{2}$ *ft*. The mass is pulled down 1 *ft* from its equilibrium position and set in motion with an initial downward velocity of 4 *ft/sec*. Suppose there is no air resistance. Find the position $u(t)$ of the mass at any time t, where $u = 0$ corresponds to the equilibrium position where the mass and spring do not move. Also find the amplitude of the spring's motion. Recall that the acceleration due to gravity is 32 ft/sec^2.

   (b) Solve the previous problem using the Laplace transform.

2. (a) A mass of weight 16 *lbs* is attached to a spring with spring constant $k = 8$ *lbs/ft*. If the mass is acted upon by an external force of $14 \sin 3t$ *lbs*, and there is no damping, find the differential equation for the position $u(t)$ of the mass at time t, where $u = 0$ corresponds to the equilibrium position. Then find the general solution of this equation.

   (b) Suppose that initially the object was given a speed of 4 *in/sec* and was placed at the equilibrium position. Solve the previous problem using the Laplace transform.

3. Transform the equation $u''' - 3u'' + 5u = 0$ into a system of first order equations. If this system is written in the form $\mathbf{x}' = A\mathbf{x}$, what is the matrix A?

4. Transform the system $\begin{cases} x_1' = 4x_1 + x_2 \\ x_2' = 3x_1 - 5x_2 \end{cases}$ into a single differential equation of second order.

5. Let A be the matrix $\begin{pmatrix} 3 & 5 \\ 4 & 4 \end{pmatrix}$.

   (a) Find the eigenvalues and associated eigenvectors of A.

   (b) Find two linearly independent solutions to the equation $\mathbf{x}' = A\mathbf{x}$.

   (c) Select one of the linearly independent solutions that you obtained in part B, and check that it actually satisfies the system $\mathbf{x}' = A\mathbf{x}$.

(d) Write a general solution and sketch the graph of possible trajectories of solutions with orientation.

(e) Find the particular solution to $\mathbf{x}' = A\mathbf{x}$ that satisfies $\mathbf{x}(0) = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

6. Let $A = \begin{pmatrix} -2 & 6 \\ -1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix}$.

(a) Find AB.

(b) Find BA.

(c) Find A^2.

(d) Is A invertible? Explain how you reach your answer.

7. Solve the following linear system of algebraic equations:
$$\begin{aligned} x_1 - x_2 - 2x_3 &= 1 \\ -x_1 + 2x_2 + 4x_3 &= 0 \\ 2x_1 + x_2 + x_3 &= 4 \end{aligned}$$

(a) Find two linearly independent real-valued solutions to the system

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \mathbf{x} \, .$$

(b) Show that the two solutions you found in part A are indeed linearly independent.

8. Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$. Find the general solution to the system $\mathbf{x}' = A\mathbf{x}$.

9. For the Laplace transform, practice problems p. 322 #1-10 and #11-23.