Derivatives Practice – Math 210/211

Power Rule: If $f(x) = ax^n$, then $f'(x) = anx^{n-1} \cdot dx$.

Note: this rule applies to radicals (fractional exponents) and simple rational expressions (negative exponents).

The "*dx*" that trails is meant to signify the chain rule. If "*x*" (the base) happens to be a function itself, then you must apply the chain rule. Technically, you always apply the chain rule, but if the base is simply "*x*", then the chain rule gives "1" as the derivative, and the answer is unchanged.

Example: If $f(x) = 4x^3$, then $f'(x) = 12x^2 \cdot 1$. The chain rule gives a "1" since it's the derivative of *x*, but we usually just write the answer as $f'(x) = 12x^2$.

Example: If $f(x) = 3(x^2+2)^5$, then $f'(x) = 15(x^2+2)^4 \cdot (2x)$. In this case, the chain rule gives a derivative of 2*x*, which trails the answer. We would normally clean up the answer and write: $f'(x) = 30x(x^2+2)^4$.

Exponential Rule: If $f(x) = e^x$, then $f'(x) = e^x \cdot dx$.

Example: $f(x) = e^{2x} \Rightarrow f'(x) = 2e^{2x}$.

Natural Logarithm Rule: If $f(x) = \ln x$, then $f'(x) = \frac{1}{x} dx = \frac{dx}{x}$.

Example: $f(x) = \ln(2x^3+x+1) \Rightarrow f'(x) = \frac{1}{2x^3+x+1}(6x^2+1) = \frac{6x^2+1}{2x^3+x+1}$.

Note how the final answer is combined into one expression for convenience.

Product Rule: If $f(x) = u \cdot v$, then $f'(x) = u \cdot v' + v \cdot u'$.

Quotient Rule: If $f(x) = \frac{u}{v}$, then $f'(x) = \frac{v \cdot u' - u \cdot v'}{v^2}$.

Practice:

1) $f(x) = x^2 + 3x + 1$

2) $f(x) = 4x^6 + 1.5x^4 + 3x^3 - x + 17$

3) $f(x) = 2(x^2 + x)^2$

4) $f(x) = \sqrt{3 + x^2}$

5) $f(x) = \sqrt[3]{x^2 + 1}$

6) $g(x) = \dfrac{3}{x^3}$

7) $g(x) = \dfrac{4}{(x^2 + 1)^2}$

8) $g(x) = \dfrac{5}{2x^4}$

9) $h(x) = e^{3x^2 + x + 2}$

10) $h(x) = (e^{2x} + 1)^4$

11) $j(x) = \ln\left(x + x^2 + 2x^3\right)$

12) $j(x) = \ln\left(e^x + 2\right)$

13) $j(x) = x \ln(x + 1)$

14) $k(x) = e^{3x} \ln x$

15) $k(x) = (\ln(x + 1) + x)^3$

16) $k(x) = \dfrac{e^{5x}}{\ln(2x)}$

17) $m(x) = \dfrac{x^2 + 1}{(x^3 + 1)^2}$

18) $m(x) = \dfrac{e^{2x} \ln x}{x^2}$

19) $n(x) = 25^2 + e$

20) $p(x) = \sqrt{\dfrac{x}{e^x}}$

Answers

(In no particular order!)

(Some have been simplified!)

$\dfrac{-4x^4 - 6x^2 + 2x}{(x^3 + 1)^3}$

$8e^{2x}(e^{2x} + 1)^3$

$\dfrac{x}{x + 1} + \ln(x + 1)$

$-\dfrac{10}{x^5}$

$2x + 3$

$-\dfrac{16x}{(x^2 + 1)^3}$

$\dfrac{e^x}{e^x + 2}$

$(6x + 1)e^{3x^2 + x + 2}$

$\dfrac{e^{3x}}{x} + 3e^{3x} \ln x$

$-\dfrac{9}{x^4}$

$\dfrac{1 + 2x + 6x^2}{x + x^2 + 2x^3}$

$\dfrac{1}{2} \cdot \sqrt{\dfrac{e^x}{x}} \left(\dfrac{1 - x}{e^x}\right)$

$\dfrac{x}{\sqrt{3 + x^2}}$

$8x^3 + 12x^2 + 4x$

0

$24x^5 + 6x^3 + 9x^2 - 1$

$\dfrac{2x}{3\sqrt[3]{(1 + x^2)^2}}$

$\dfrac{3(x + 2)(\ln(x + 1) + x)^2}{x + 1}$

$\dfrac{5xe^{5x} \ln(2x) - e^{5x}}{x(\ln(2x))^2}$

Any errors (real or imagined), contact surgent@asu.edu. Good luck! (Updated 8/26/09)