

Projection Operator Method for Quantum Groups

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Abstract

A method of projection operators is developed for quantum groups. Here the term “quantum groups” means q -deformed universal enveloping algebras of the Kac-Moody (super)algebras of finite growth (these (super)algebras include all finite-dimensional simple Lie algebras and classical superalgebras, infinite-dimensional affine (loop) Lie algebras and superalgebras). At first we describe combinatorial structure of root systems of the Kac-Moody (super)algebras of finite growth. The result is used for construction of a q -analog of the Cartan-Weyl basis and for description of explicit structure of the universal R-matrices and the extremal projectors.

Then we consider some applications of the extremal projectors. In particular, we apply them for explicit description of reduction algebra over quantum algebras $U_q(su(n))$ and for construction of a q -analog of the Gel'fand-Tsetlin basis for $U_q(su(n))$. We also use the projector operator method for construction of the theory of Clebsch-Gordan coefficients for quantum algebras $U_q(su(2))$ and $U_q(su(3))$. In particular, we obtain a very compact general formula for the canonical $U_q(su(3)) \supset U_q(su(2))$ Clebsch-Gordan coefficients in terms of the $U_q(su(2))$ Wigner 3nj-symbols which are connected with the basic hypergeometric series. Finally we introduce some generalizations of extremal projector for the case of the quantum algebra $U_q(su(2))$ and consider their applications.