

TWO PROOFS OF THE EVERITT-MARIĆ
RESULT FOR THE LEGENDRE DIFFERENTIAL EQUATION

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Abstract

This talk is joint work with W. N. Everitt (University of Birmingham, Birmingham, England), Jorge Arvesú and Francisco Marcellán (Carlos III University, Madrid, Spain).

The spectral analysis of the classical, formally symmetric, Legendre differential equation

$$\ell[y](x) := (1 - x^2)y'' - 2xy' + ky = \lambda y \quad (x \in (-1, 1)),$$

where k is a fixed, non-negative parameter and λ is a complex eigenvalue parameter, was first studied in detail by E. C. Titchmarsh in 1940. In particular, it is well known that the differential operator $A : \mathcal{D}(A) \subset L^2(-1, 1) \rightarrow L^2(-1, 1)$, defined by

$$\begin{aligned} Af &= \ell[f] \\ f &\in \mathcal{D}(A), \end{aligned}$$

where

$$\begin{aligned} \mathcal{D}(A) &= \{f : (-1, 1) \rightarrow \mathbb{C} \mid f, f' \in AC_{loc}(-1, 1); \\ &\quad f, \ell[f] \in L^2(-1, 1); \lim_{x \rightarrow \pm 1} (1 - x^2)f'(x) = 0\}, \end{aligned}$$

is self-adjoint with discrete spectrum $\sigma(A) = \{n(n+1) + k \mid n \in \mathbb{N}_0\}$ and has the Legendre polynomials $\{P_n(x)\}_{n=0}^\infty$ as a (complete) set of eigenfunctions. In a recent result by Everitt and Marić, these authors proved the following (optimal) smoothness result for $\mathcal{D}(A)$:

$$f \in \mathcal{D}(A) \Rightarrow f' \in L^2(-1, 1).$$

In this talk, we will give two proofs of this result. The first proof is the one given by Everitt and Marić, using the Chisholm-Everitt inequality. The second proof is due to Arvesú, Littlejohn, and Marcellán using the general left-definite theory recently developed by Littlejohn and R. Wellman.