

MAT 270 – Calculus I
Sample Test 3

NAME: _____

ASU ID # _____ Section No. _____

Note: Read carefully! Closed book, closed notes and no calculators. All work to be shown on the pages provided. (Use reverse side if necessary).

1. Find $f'(x)$ for each of the following: (1 points each)
 - (a) $f(x) = \sin(1 + x^2)$
 - (b) $f(x) = e^{x+\ln(x)}$
 - (c) $f(x) = \ln(\sin(x))$
 - (d) $f(x) = \ln(\ln(\sin(x)))$
 - (e) $f(x) = x^{x^2}$
2. (2 points each) If $e^y = xy$ find
 - (a) $dy/dx =$
 - (b) $d^2y/dx^2 =$
3. Find two points on the hyperbola $x^2 - y^2 = 4$ where the tangent line passes through the point $(1, 0)$. (4 points)
4. Recall that $\cosh(x) = (e^x + e^{-x})/2$, $\sinh(x) = (e^x - e^{-x})/2$ and $\tanh(x) = \sinh(x)/\cosh(x)$. (2 points each)
 - (a) Find $\lim_{x \rightarrow 0} \tanh(x)$
 - (b) Find $\lim_{x \rightarrow \infty} \tanh(x)$
 - (c) Solve for x where $1 + \sinh(x) = \cosh(x)$
5. Suppose that the radius of a circle is increasing at a rate of 2m/s. How fast is the area of the circle increasing when the radius is 4 m? (4 points)
6. Suppose a balloon is released from a point 100m from an stationary observer, ascends straight upward at a rate of 5 m/s as shown in figure 1. After 20 seconds, how fast is the distance from the observer to the balloon increasing? After 20 seconds, how fast is the angle increasing? (8 points)
7. Find the standard linear approximation to the function $f(x) = e^x$ near the point $x = 0$. (2 points)
8. Using the standard linear approximation to the function $f(x) = e^x$ near $x = 0$ estimate $e^{0.5}$. (2 points)
9. Use the standard linear approximation to estimate $24^{1/2}$. (2 points)

Figure 1: Piecewise function of problem 6

Figure 2: Piecewise function of problem 12

10. Find the absolute maximum and minimum values of the function $f(x) = x^2(1 - x)$ for the interval $-1 \leq x \leq 1$. (4 points)
11. Sketch the graph of the following function indicating: critical numbers, local maximum and minimum, inflection points and asymptotes if any. $f(x) = xe^{-x^2/2}$. You can use 0.37 for e^{-1} , 0.22 for $e^{-3/2}$ and 1.73 for $e^{1/2}$. (4 points)
12. Find the area of the biggest rectangle that can be inscribed as shown in figure 2 into the right triangle. (4 points)