Postprocessing of nonuniform MRI

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Outline

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Postprocessing of image data for

- Deblurring
  - Removal of potential filtering effects introduced by uniform sampling
  - Extend for spatially variant effects introduced by regridding
- Edge Detection
  - At the reconstruction stage identifying edges improves segmentation
  - Extend ideas for nonuniform gridding
  - Initial work in the image domain
  - A new regularization based on **edge concentration kernels**.

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It is well known that Fourier reconstruction

- introduces Gibbs ringing at tissue boundaries - seen as oscillations in one dimensional cuts
- but that filtering alleviates ringing, while introducing blurring.

Accurate segmentation and registration relies on *accurate* images.
Methods for uniform data at reconstruction stage include

- Gegenbauer reconstruction (Gelb et al)
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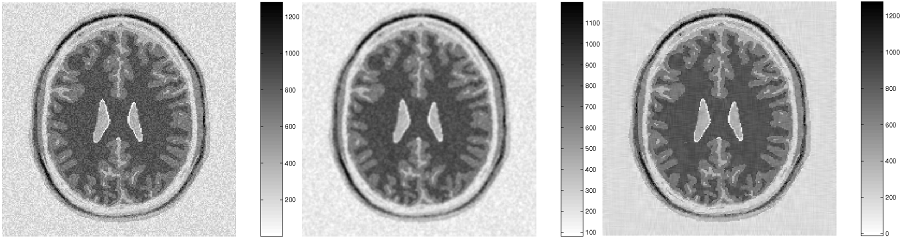
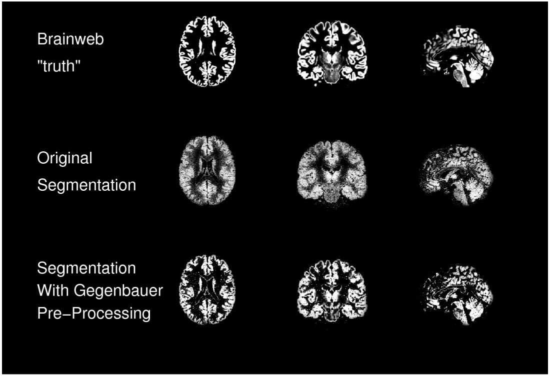
Example Gegenbauer Concentration Method on Fourier Data (no noise)



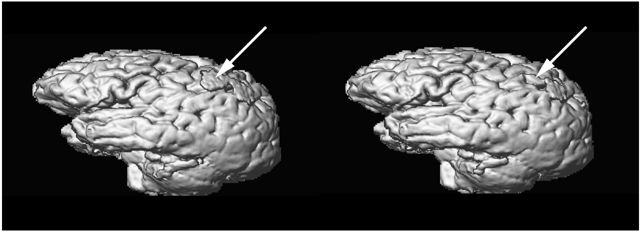
Left to Right: (a) A slice of the MRI reference data from McGill sampled on a $[218 \times 218]$ *grid (b) Filtered reconstruction.*
(c) Reconstruction of the image using the concentration edge detection method and Gegenbauer reconstruction.

Example: Improvement of image quality for segmentation

Archibald, Chen, Gelb, & Renaut



Gray matter segmented probability maps for the original and **Gegenbauer image reconstruction with edge detection** of a particular randomly generated MNI digital brain phantom with a 9% level of noise and 40% intensity non-uniformity.



SPM brain extraction of one particular subject, where the picture on the right used edge detection and Gegenbauer reconstruction as a pre-segmentation step.

Current Gegenbauer edge concentration method is most easily implemented in the Fourier data.

- It does increase contrast in images, ie sharpens edges.
- Not immediately extendible for non uniform data in Fourier domain from modified acquisition schemes.
- But concentration edge detection can be extended for pixel data emphasizing edge detection.

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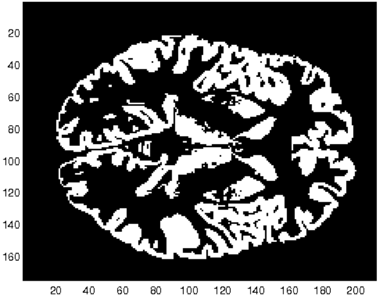

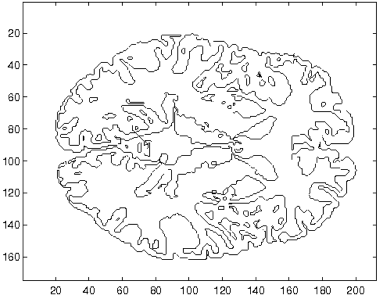

Left to right (a) Simulated MRI brain. (b) Final segmentation result showing all open and closed contours obtained from concentration method with segmentation.

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Extensions for blurred and noisy data in the pixel data- approach usually

- Deblur with regularization for handling ill-posed problem
- Apply edge detection to obtained solution

Invariant Point Spread Function:

$$G = HF + N$$

is matrix formulation of convolution $g = h \star f + n$, *for noise* n, *image* g *and PSF* h, *required image* f. *Finding solution* F *requires regularization, which can impose sparsity condition on edges in image through regularization* R

$$\hat{F} = \arg\min_F \{ \|HF - G\|_2^2 + \lambda R(F) \}$$

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Errors in the model (Nonuniform Point Spread Function)

- PSF is not invariant, particularly for new acquisition processes.
- Need to modify the restoration model

(Total Least Squares)

Assume some information about the PSF, and solve for (E, F)

$$G = (H + E)F + N/\eta$$

where H *is an approximate PSF,* E *is unknown error, and* η *a scaling on noise and error. Implies Rayleigh Quotient form of Total least squares:*

$$\hat{F} = \arg\min_F \Phi = \arg\min_F \frac{\|HF - G\|_2^2}{1 + \eta^2\|F\|_2^2}$$

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Regularization term R : Total Variation

$$R(x) = TV(x) = \|Lx\|_1,$$

L is discrete differentiation operator usually first order: $Lx_i = x_{i+1} - x_i$

Using optimization code requiring the gradient of $R(F)$, note $\|\cdot\|_1$ is not differentiable and is replaced by relaxed form, with small β,

$$\|Lx\|_1 \approx \|Lx\|_\beta = \sum_i \sqrt{(Lx)_i^2 + \beta^2}$$

Otherwise use convex programming tool Matlab
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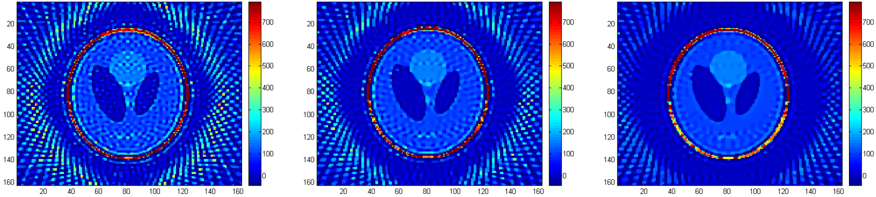
Example of TV Regularization with Spatially Variant PSF

Use Total Least Squares - imposes some structure in following example - Toeplitz.



Shepp Logan phantom blurred and noise in the sinogram. Reconstructed using filtered back projection. Phantom deblurred using a wider Gaussian PSF. (a) deblurring using TV regularized LS, (b) intermediate weighted TVTLS and (c) TVTLS weight 1.

But TV introduces artifacts: solution is blocky and not smooth

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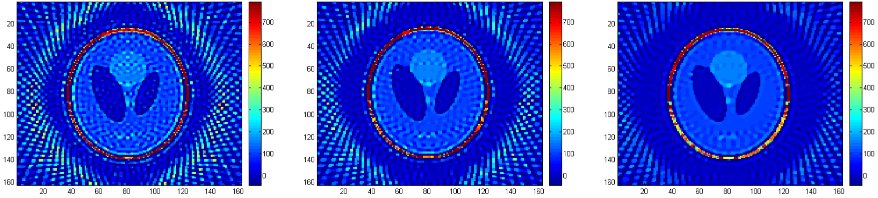

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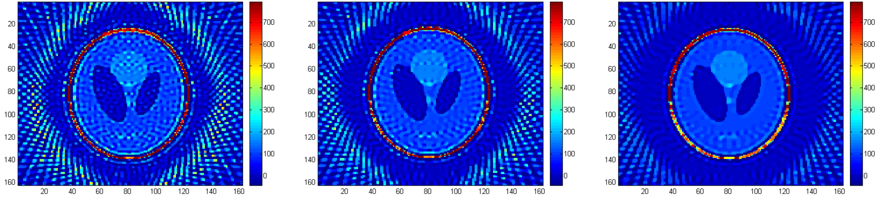

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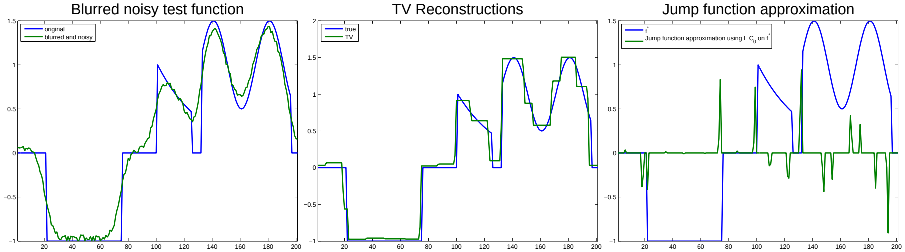
Restoration with TV regularization for blurred and noisy signal



(a) Gaussian PSF and added noise. (b) TV reconstruction is piece wise constant, stair case effect is typical. (c) Plot of the regularization term $\|Lf\|_\beta$ TV solution is sparse after applying the differentiation operator L.

Remark

TV function is a jump function: Is it appropriate? What about the jumps at wrong locations

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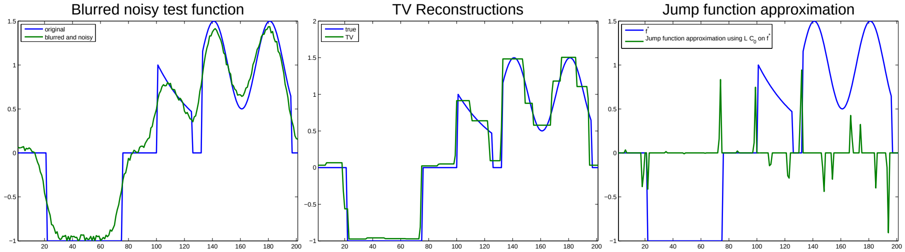

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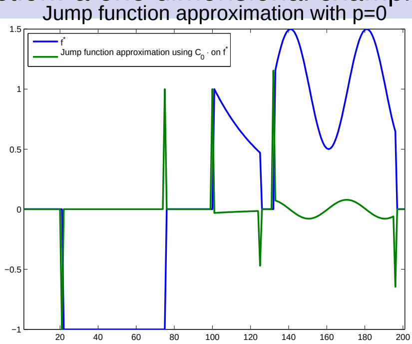
The exact jump function: a one dimensional example



- *Piecewise smooth test function, several jump discontinuities. (no blurring or noise).*
- *Blue is the original signal f.*
- *Green plot of the function Lf, shows jumps at the edges, but non-zero in the smooth parts.*
- *Green is the function minimized in regularization.*
- *Hence regularization minimizes over the smooth region also.*

(Heuristics)

- *Linear translation invariant operator approximating jumps*

$$[f](x) = f(x^+) - f(x^-)$$

  in function f, where x^+ is right hand limit at jump and x^- the left hand limit.
- *The concentration operator, concentrates near jump discontinuities, separated from smooth regions where $[f](x) = 0$. Contrast with TV which concentrates at jumps but is nonzero also in smooth regions.*
- *Effectiveness depends on choice of concentration factors $\sigma(\cdot)$*
- *Developed in Fourier domain, but implementable on spatial data.*

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The concentration function: (for f piecewise smooth 2π periodic function equally sampled at $2N+1$ grid points on $[0, 2\pi - \Delta x]$).

$$\tilde{T}_N^{\tau}[f](x) = \Delta x \sum_{j=0}^{N} f(x_j) \sum_{k=1}^{N} \sigma(\frac{k\Delta x}{\pi}) \frac{\sin(k\Delta x/2)}{k\Delta x/2} \sin k(x - x_j).$$

This is a discrete convolution in Fourier domain for pseudospectral coefficients

$$\tilde{f} = \frac{1}{2N+1} \sum_{j=1}^{2} f(x_j) e^{-ikx_j}$$

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(Spatial Data)

Applied between grid points at $x_{j+1/2} = (2\pi(j+1/2))/(2N+1)$ *yields matrix operator*

$$C_{i,j}(\sigma) = \Delta x \sum_{k=1}^{N} \sigma\left(\frac{k\Delta x}{\pi}\right) \frac{\sin(k\Delta x/2)}{k\Delta x/2} \sin k \frac{2\pi(i+1/2-j)}{2N+1}.$$

Example trigonometric concentration factor

$$\sigma_{2p+1}(s) = c_p 2^{2p} s \sin^{2p}(\frac{\pi s}{2}),\, c_p = \frac{\sqrt{\pi}}{2^{2p}} \frac{\Gamma(2p+1)}{\Gamma(\frac{2p+1}{2})}.$$

It yields operator C_p *of order p, which determines locality of the detector.*

- *Indeed, the case* $p=0$ *corresponds to the first order difference* $f_{j+1} - f_j$*, which is just the TV operator*

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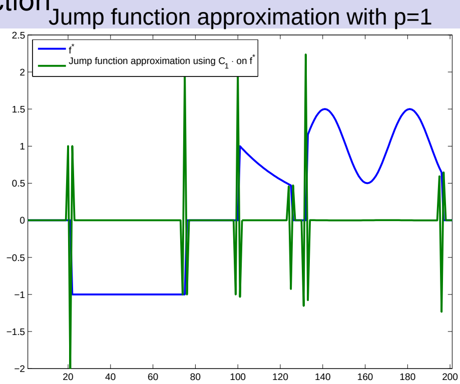
A new Jump Function



Notice improvement of the jump function in smooth regions but experiences ringing artifacts at the edges.

$p = 1$ *yields the third order backward difference operator*

$$\triangle^3 f_{j+1/2} := -f_{j+1} + 3f_{j+1} - 3f_j + f_{j-1} \approx -(\Delta x)^3 f_j'''$$

Larger p increases order of operator, impacting locality of detector.

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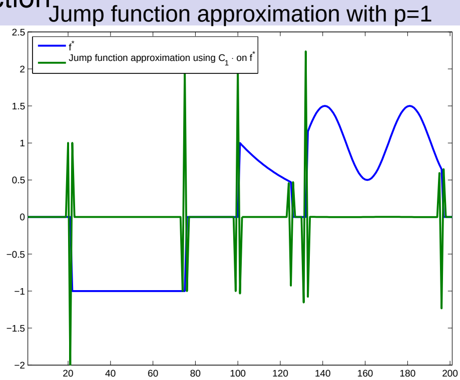

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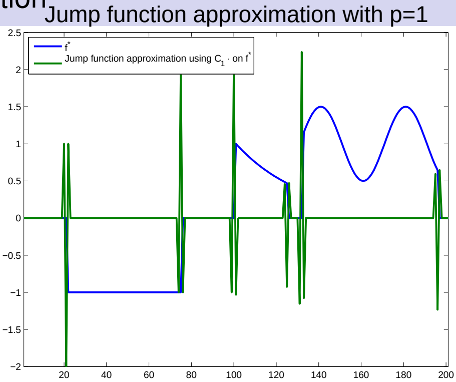

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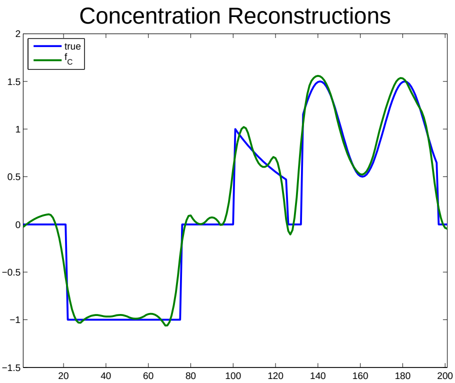
(Heuristics)

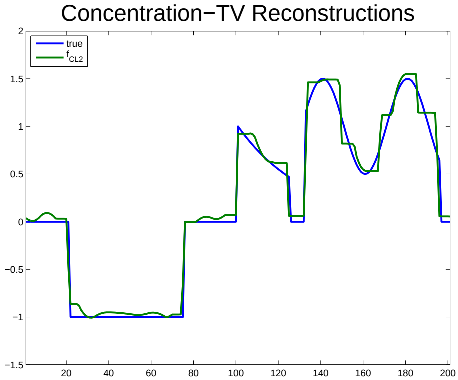
- *Use for the regularization term* $R(F)$, $\|C_pF\|_\beta$ *instead of* $TV(F)$.
- *Estimate initial location of edges with* $p = 1$, *more robust than* $p = 0$..
- *Define distance and use TV at the edges and* $p = 1$ *away from edges.*
- *Chose regularization parameter based on* $R(F) = R(F_{exact})$! *Well assume some known information on the signal always occurs.*
- *Make the method adaptive- initially estimate jumps and refine using thresholding.*

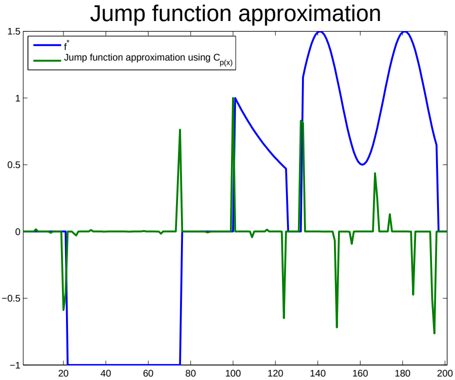
Regularization using the Concentration Function: Example



Using regularization with $p = 1$ in $\|C_p F\|_\beta$ and the resulting jump function on the right. It has fewer false positives than the TV in the smooth region.

Adaptivity improves the reconstruction





Reconstruction of the signal by regularization with $\|C_p f\|_\beta$, adaptive p (a) The jumps are still at the right position and the smooth regions are now better represented. (b) Shows the new jump function $C_p f$ with even fewer false positives compared to non adaptive regularization.

Future Work

- Extend to two dimensions
- Include deblurring for spatially variant PSF
- Examine whether method can be applied in the Fourier domain, namely no reconstruction for nonuniform data.

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- R. Archibald and A. Gelb, A method to reduce the Gibbs ringing artifact in MRI scans while keeping tissue boundary integrity, IEEE Trans. Medical Imaging, 21(2002), 305–319.
- R. Archibald and A. Gelb, Reducing the effects of noise in image reconstruction, J. Sci. Comput., 17(2002), 167–180.
- R. Archibald, K. Chen, A. Gelb and R. Renaut, Improving Tissue Segmentation of Human Brain MRI Through Pre-Processing by the Gegenbauer Reconstruction Method, NeuroImage, 20(2003), 489–502.
- R. Archibald, A. Gelb and J. Yoon, Polynomial Fitting for Edge Detection in Irregularly Sampled Signals and Images, SIAM J. Numer. Anal. 43(2005), 1, 259–279.
- A. Gelb and E. Tadmor Adaptive Edge Detectors for Piecewise Smooth Data Based on the minmod Limiter, Journal of Scientific Computing, 28(2006), 279–306.
- L. Rudin, S. Osher and E. Fatemi, Nonlinear total variation based noise removal algorithms, Physica D, 60(1992), 259–268.