Information Extraction from PET Images

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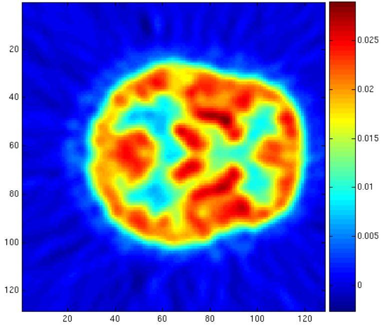
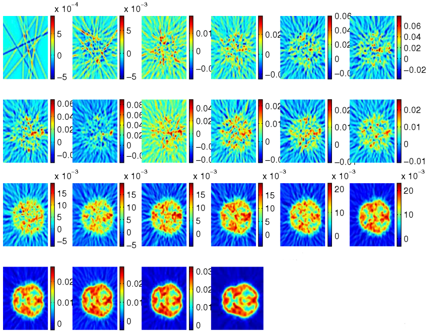
Example of typical PET scan



Typical PET Images show

- High noise content (non Gaussian)
- High blurring
- Reconstruction artifacts
- Reconstruction using filtered backprojection

Dynamic PET scan



Dynamic data

- Very poor initial scans
- Noise levels change across scans
- Solve inverse problem to estimate kinetic parameters.
- What does it mean to improve these images?

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- $\Rightarrow$ **use imaging**

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- **Deblur images by deconvolution**

Inverse Problem

- Find f from $\boxed{g = f * h + n}$ given g and h with unknown n.
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- Assuming normal distributed n yields including regularization

$$\hat{f} = \arg\min_{f}\{\|g - f * h\|_2^2 + \lambda R(f)\}$$

Regularization Methods (review)

- Common methods are Tikhonov (TK).

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- Sparse deconvolution (L^1) (not relevant for PET images)

$$R(f) = \|f\|_1 = \int_\Omega |f(x)| dx.$$

Regularization Review

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- λ **Governs the trade off** between the fit to the data and the smoothness of the reconstruction and can be picked by the L-curve approach, (see Hansen, Inverse Problems)

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- **L1 yields spike trains**

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- The TV objective function is **non differentiable**

$$J(f) = \|g - Hf\|_2^2 + \lambda\|\nabla f\|_1$$

Differentiability of TV - $1D$ (tensor product in $2D$)

$$R(f) = \sum_i \|f_{i+1} - f_i\|$$

- for a small β define

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- choose β in 10^{-5} to 10^{-9}

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- **Evaluation** of the OF and its gradient **is cheap** (some FFTs and sparse matrix-vector multiplications)
- Problems are usually large and many iterations are needed.

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- Estimates exist for PET scanners from phantom scans
- PSF is spatially variant and also depends on the scanned object
- Hence even if provided PSF is always **only an estimate**
- for the PET scans presented here, a $6mm$ half width Gaussian was assumed.

Simulated PET

- On the left simulated PET from blurred segmented MRI scan using **Gaussian PSF** and noise added.

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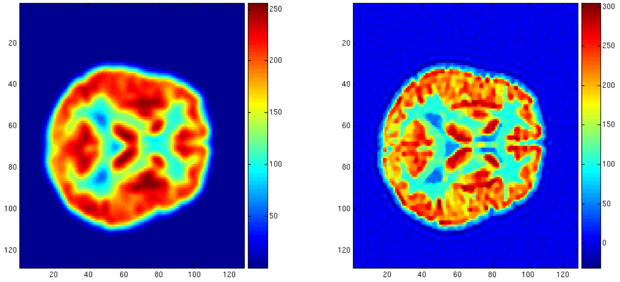
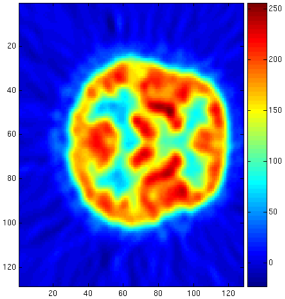
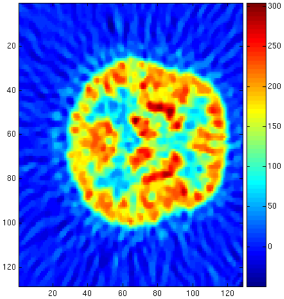
- On the right deblurred PET with TV and known PSF.



Recover real PET image





- Reconstruction done using Filtered Back Projection
- PSF estimated by a Gaussian
- TV regularization

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Total least squares (TLS)

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- f_{TLS} can be found from SVD of $[H, g]$ (Golub et al)

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- p=2 e.g. Golub et al (1999), Renaut et al (2005)

Scaled TLS: different noise levels

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- γ accounts for different noise levels in H and g.

Scaled Total Least Squares with Regularization

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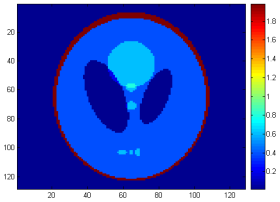
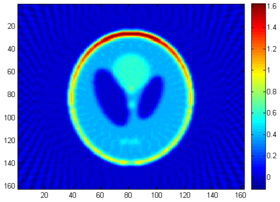
- Which is greater , the error in the PSF or the error in the measured data?

Test Problem Noisy Shepp Logan Phantom

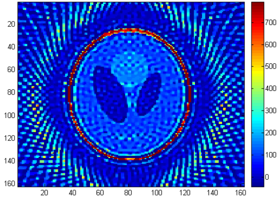
- Use 128x128 Shepp Logan Phantom
  - Blur with Gaussian $h(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{|x|^2}{2\sigma^2}}$ with $\sigma = 1.5$ (6mm half width)
  - Take forward Radon transform with 45 angles
  - Add Poisson Noise to sinogram
  - Transform back, with filtered back projection





Deconvolving the Shepp-Logan Phantom

- Gauss PSF with $\sigma = 2$ and TV regularization



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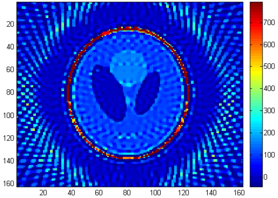
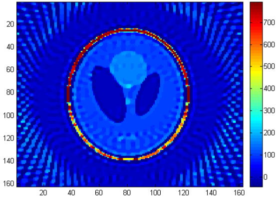


$\gamma^2 = 7.3e - 9$



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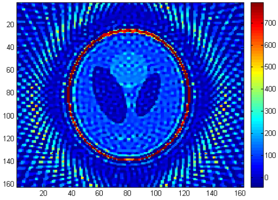

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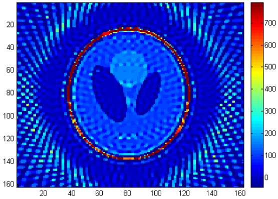

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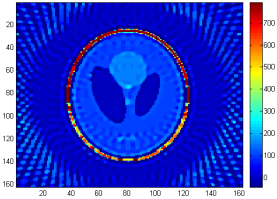

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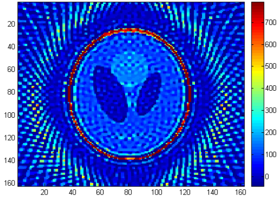

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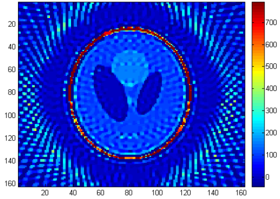

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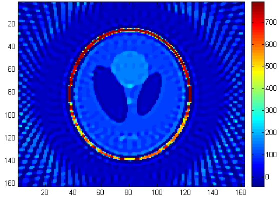

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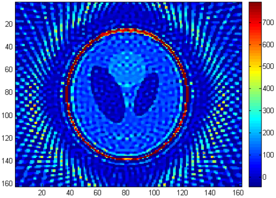

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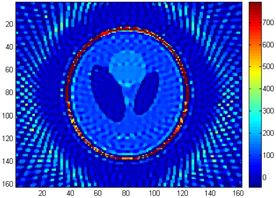

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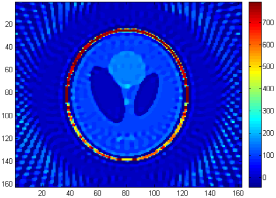

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- Scaling shows how to improve impact of badly chosen PSF.
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- For scaled problem γ^2 is 1, 50, resp.

Real PET data

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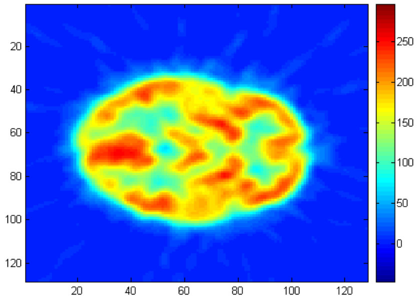
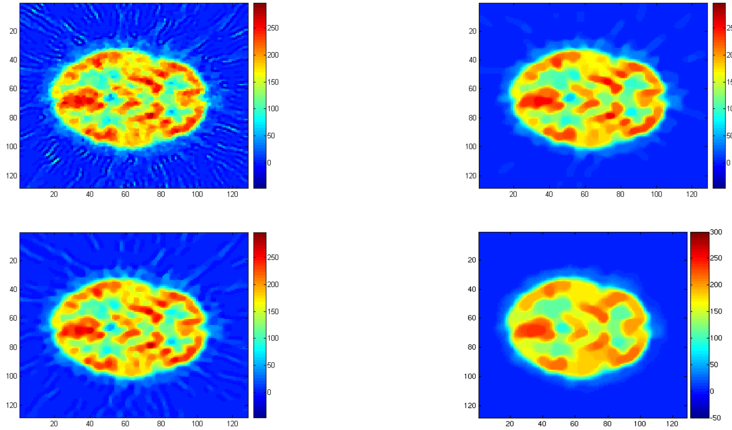
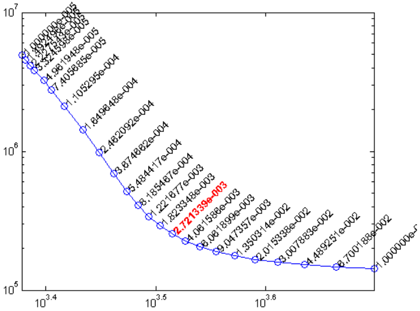


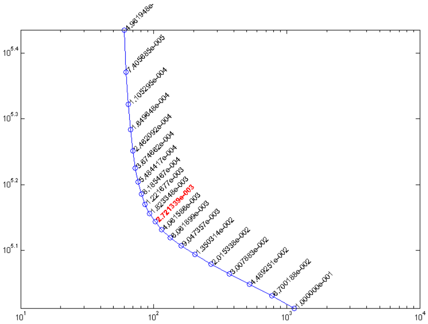
Numerical Results Scaled RTVTLS

PSF $6mm$ half width Gaussian, $\gamma = 0$ (LS), $3.7e-7$, $1e-5$ and $1e-4$ (top to bottom and left to right 0, 50, $1.4e3$, $1.4e4$)



L-curve





- Notice optimal λ by L-curve is similar for LS and Scaled TLS.

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  allows further tuning in case of an unknown PSF
- Iterations are expensive.

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- What can be achieved with wavelets- see Wolfgang Stefan pm.

Acknowledgments

- Hongbin Guo (Total Least Squares)
- Haewon Nam and Kewei Chen for the data and discussions on PET imaging
- Supported by: Arizona Alzheimer's Research Center and NIH NIBIB