Signal restoration through deconvolution applied to deep mantle seismic probes

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Abstract

In [4] we present a method of signal restoration to improve the signal to noise ratio, sharpen seismic arrival onset, and act as an empirical source deconvolution of specific seismic arrivals. The method is used on the shear wave time window containing SKS and S, whereby using a Gaussian PSF produces more impulsive, narrower, signals in the wave train. The resulting restored time series facilitates more accurate and objective relative travel time estimation of the individual seismic arrivals. Clean and sharp reconstructions are obtained with real data, even for signals with relatively high noise content. Reconstructed signals are simpler, more impulsive, and narrower, which allows highlighting of some details of arrivals that are not readily apparent in raw waveforms.

The Problem

- Earth's interior is inaccessible
- Remote sampling is required
- Important steps include
  - **(a)** **accurate characterization** of seismic energy
  - **(b)** reliable estimation or measurement of seismic **wave timing**

To improve (a) and (b) over traditional methods we use a deconvolution-based restoration method that deblurs a given signal, sharpens seismic signal onsets and improving the visibility of emerging secondary seismic arrivals.
This is demonstrated on

(a) the signal restoration of SKS and S (or S_{diff}) in synthetic seismograms and

(b) the restoration of actual data for 31 seismic recordings of a deep focus South American earthquake.

Deconvolved seismograms are used for

- Relative travel time determination
- Waveform distortion diagnostics
- Demonstration of the detection of two very similar shaped pulses nearly superposed in time

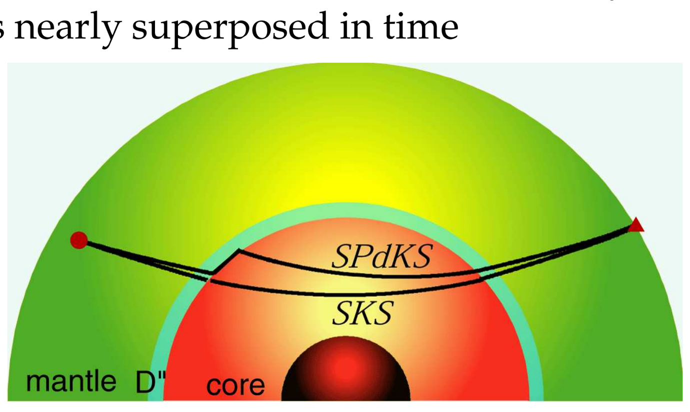

Figure 1. *SKS and SPdKS is blurred by Earth's mantle.*

Signal degradation

Signal degradation in this study is modeled by a convolution + noise [5]

$$g = f * h + n, \tag{1}$$

with a Gaussian point spread function (PSF)

$$h(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}, \tag{2}$$

where the parameter σ governs the width of the function.

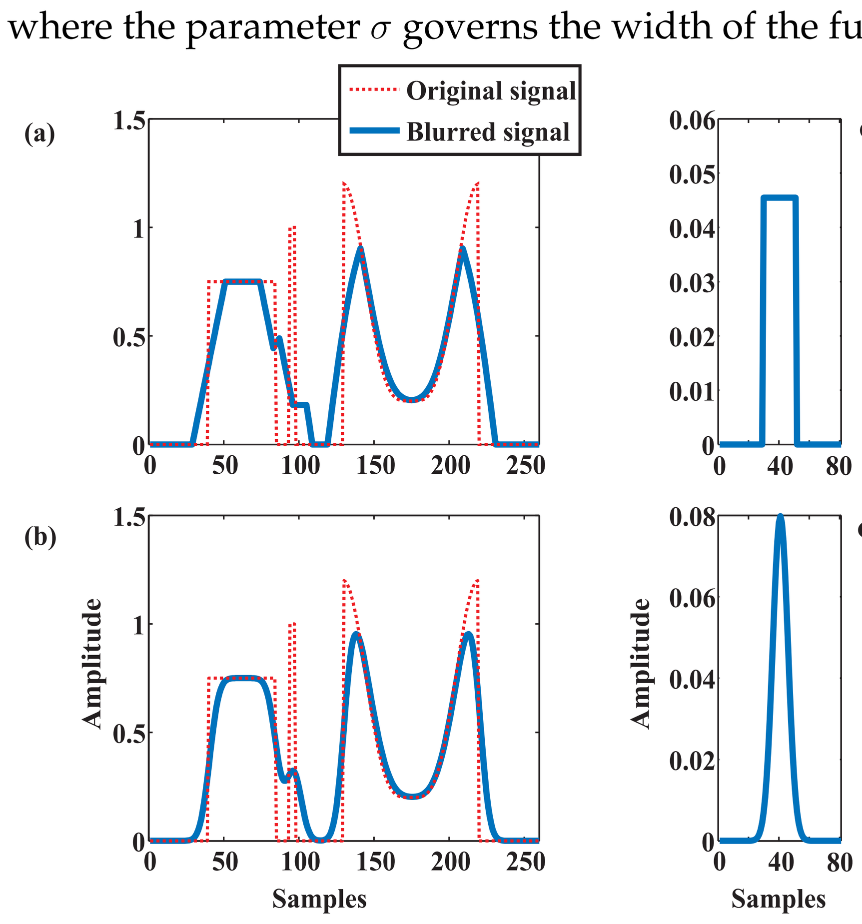

Figure 2. *Blurring by different PSFs.*

Signal restoration

The goal is to identify the original signal f, given the observed signal g in (1). This inverse problem is **badly ill posed** i.e. a regularization technique has to be employed. Including regularization we solve the inverse problem by [5]

$$\hat{f} = \arg\min_f \{\|g - f * h\|_2^2 + \lambda R(f)\}, \tag{3}$$

in which the second term $R(f)$ is the regularization term.

- λ Governs the trade off between the fit to the data and the smoothness of the reconstruction and can be picked by the L-curve approach
- Common regularization methods are Tikhonov and Total Variation (TV) regularization
- L^1 regularization 1st used for seismograms in [1], see also [2]
- TV yields a piece wise constant reconstruction and preserves the edges of the signal
- Tikhonov yields a smooth reconstruction
- For our application we are interested in a sharp reconstruction, therefore TV is the better choice
- To find the minimum of (3) we use a limited memory BFGS method

The TV of a function f is defined by

$$\mathrm{TV}(f) = \int_{-\infty}^{\infty} |f'(t)|\, dt. \tag{4}$$

Experiments

We demonstrate the deconvolution method with edge detection on synthetic and real seismic data.

Synthetic seismograms

The synthetic seismograms are produced using the 1-D PREM reference model.

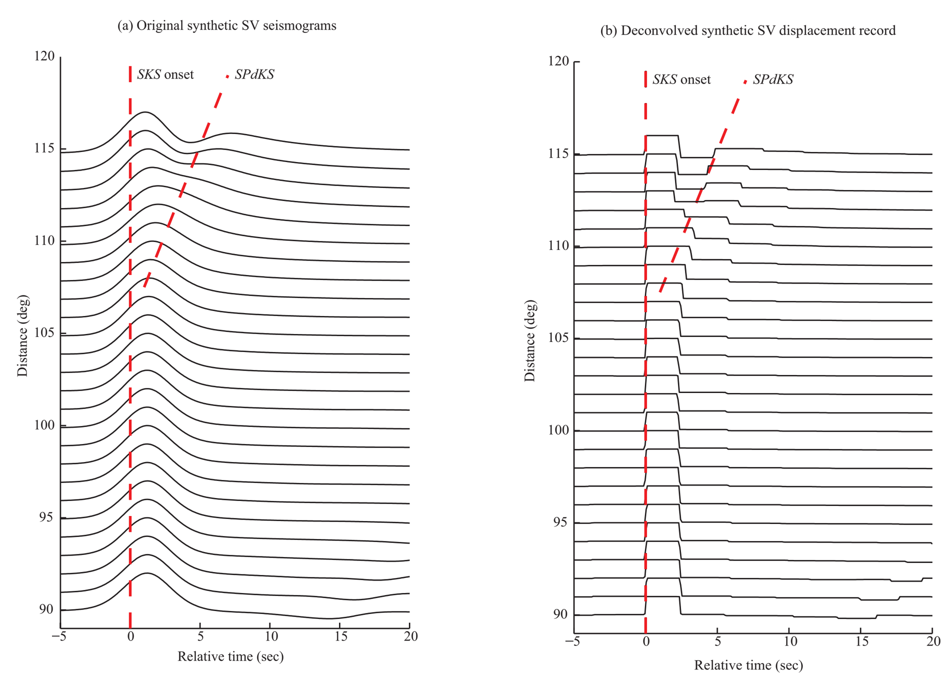

Figure 3. *Original and deconvolved SV trace, aligned at edges in (b)*

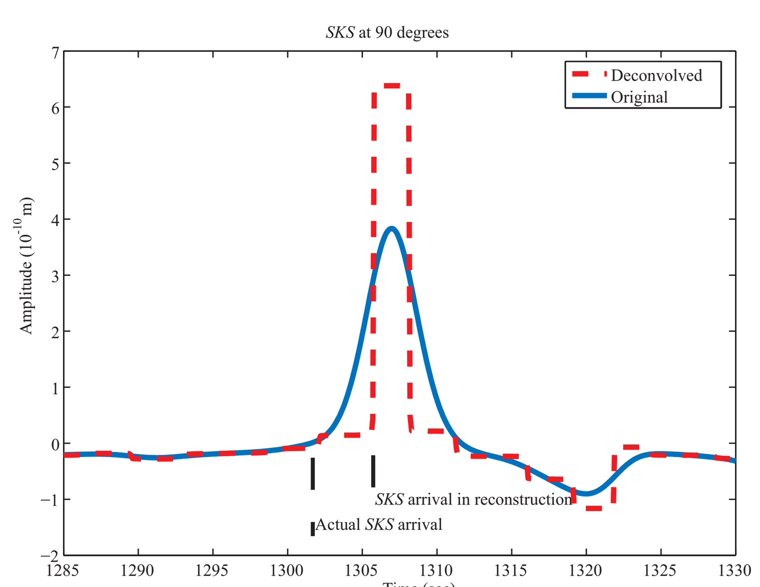

Figure 4. *Deconvolved synthetic SKS*

Real seismic data

In the following we apply our method to the records of an earthquake in South America $M_b = 7.2$ on May 12, 2000, recorded at 31 broadband stations in Europe.

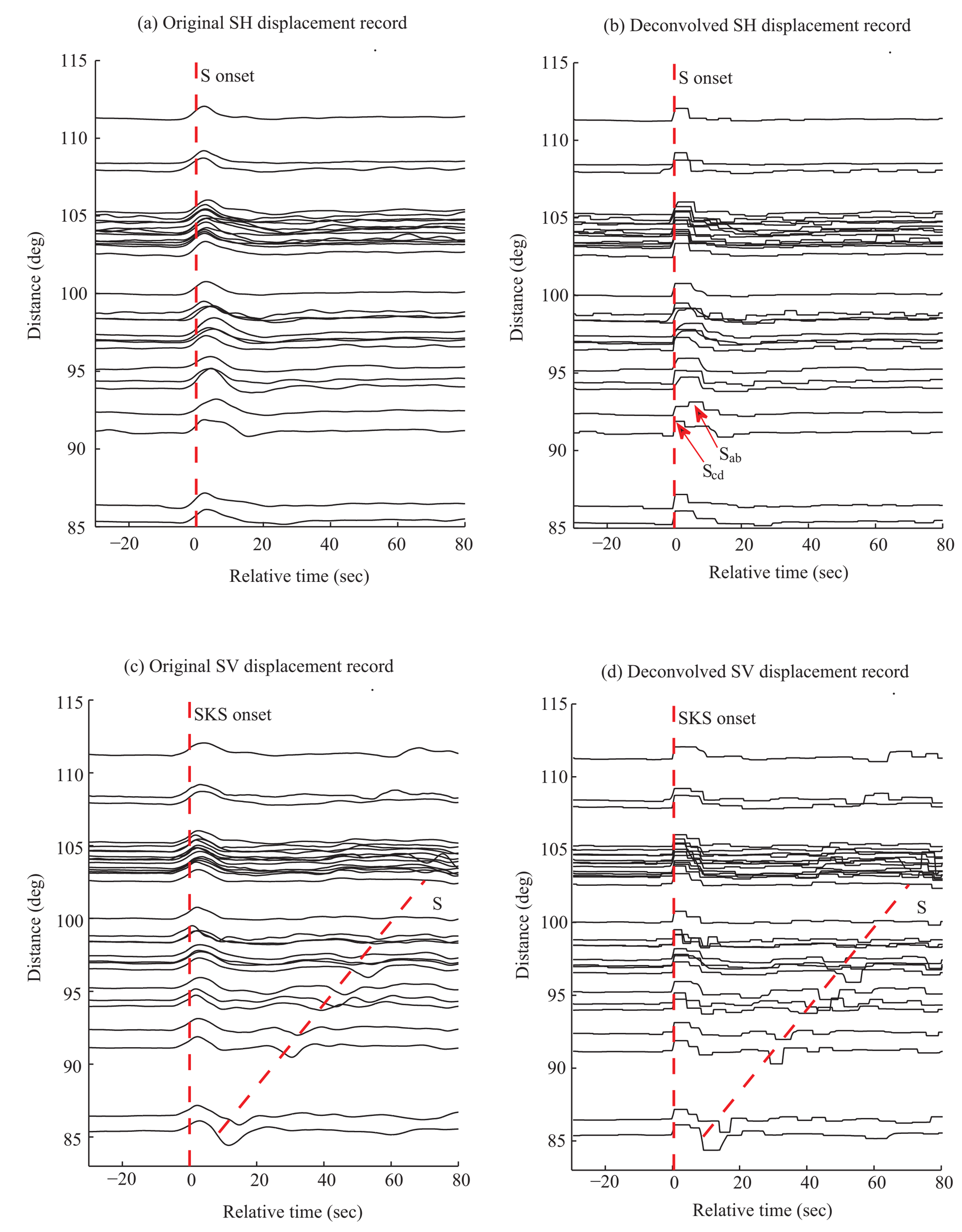

Figure 5. *Original and deconvolved SH and SV trace. Broadening between 90 and 98 deg is consistent with the D" related S_{ab} and S_{cd}*

Conclusions

We showed that TV-regularized deconvolution results in

(a) sharp and clear reconstructions of both noise-free synthetic seismograms and noise-contaminated real seismograms of a test case,

(b) the signal reconstruction algorithm resulted in more accurate relative timing and amplitude information from the deconvolved traces than is presently possible with the raw traces.

Future work will include

- verify results with more synthetic and real data
- use signal/noise separation techniques as pre-processing like in [3] to improve the signal-to-noise ratio.

Acknowledgment

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References

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