

MAT 473 Intermediate Real Analysis II

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Spring 2009

revised March 20, 2009

Integrating nonnegative functions — Exercises

1. Let $f \in L^+(\mathbb{R})$ and $c \in \mathbb{R}$. Prove that

$$\int_{\mathbb{R}} f(x+c) dx = \int_{\mathbb{R}} f.$$

Hint: first do it for simple functions, then use the Monotone Convergence Theorem. Note that integrals over \mathbb{R} mean (by default) with respect to Lebesgue measure.

2. (Borel-Cantelli Lemma) Let $\{A_n\}$ be a sequence of measurable sets in a measure space (X, μ) , and suppose that $\sum_1^\infty \mu(A_n) < \infty$. Prove that almost every point is in only finitely many A_n 's. That is, show that there exists $B \subset X$ such that $m(B) = 0$ and for all $x \notin B$, the set $\{n \in \mathbb{N} : x \in A_n\}$ is finite. This exercise occurred already in the section on measures, but now do it by applying the Monotone Convergence Theorem to the functions χ_{A_n} .