

# MAT 473 Intermediate Real Analysis II

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## Measurable functions — Exercises

1. Let  $U \subset \mathbb{R}$  be open and  $f : U \rightarrow \mathbb{R}$  be differentiable. Prove that  $f'$  is measurable.
2. Prove that every Lebesgue measurable function on  $\mathbb{R}$  is a.e. equal to a Borel function. Hint: do it first for simple functions.
3. Let  $g : [0, 1] \rightarrow [0, 2]$  be the modified Cantor function. Prove that  $g(C)$  has measure 1.