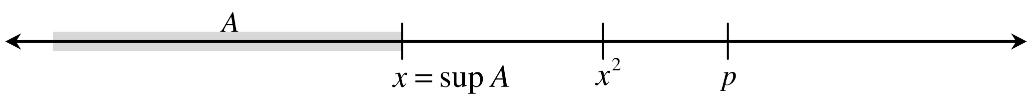
Theorem 0.23 Visualization

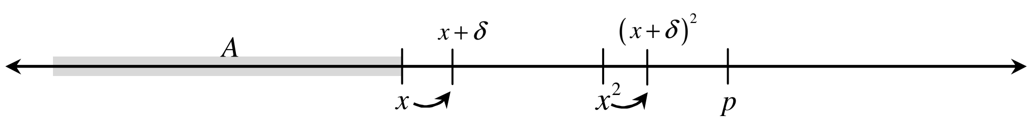
The following diagrams should help you visualize the reasoning for the proof of Theorem 0.23. See #7 a and b in the Section 0.5 Study Guide to get you started.

Part 7c of the study guide is about ruling out the case $x^2 < p$. To visualize what is going on here, sketch the diagram:



Notice that the condition $x^2 < p$ ensures there is space to the right of x^2 but left of *p* for there to be another square. If we can squeeze something in there, it will give us an element of *A* that is larger than *x*. But that would contradict *x* being an upper bound of *A* and therefore rule out the case $x^2 < p$.

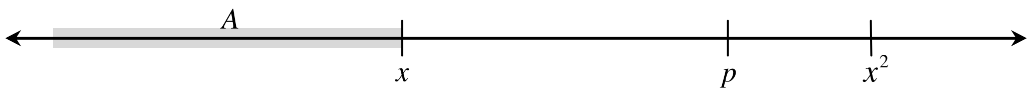
To accomplish this formally, what we want to do is add an amount $\delta > 0$ to *x* that is small enough to keep the square smaller than *p* as illustrated below:



So the inequality we would like to achieve is $(x+\delta)^2 \le p$. As shown in the diagram, this gives us the contradiction of $x+\delta$ being an element of *A* on the one hand but larger than *x* on the other.

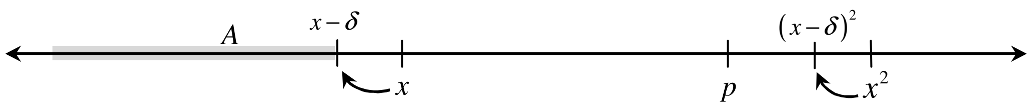
The remainder of Part 7c in the Study Guide will help you understand how δ is chosen to establish this condition in the proof.

Part 7d of the study guide is about ruling out the case $x^2 > p$. To visualize what is going on here, again sketch the diagram:



In this case, there is space to the left of x^2 but to the right of p for us to insert another square. This would generate an upper bound for A that is smaller than x. Since that contradicts x being the *least* upper bound of A, it rules out the case $x^2 > p$.

To accomplish this formally, we need to subtract an amount $\delta > 0$ from x that is small enough to keep the square bigger than p as illustrated below.



So the inequality we would like to achieve is $(x-\delta)^2 \geq p$.

As shown in the diagram, this would have to push A back some since $x-\delta$ would have to be an upper bound. Since $x-\delta$ is smaller than x, this contradicts x being the *least* upper bound.