

MAT 371
Sections 4.3 – 4.4 Study Guide

1. Read the definition of *relative maximum (minimum)* on p. 119. Why is the neighborhood Q important in this definition? How does the use of the neighborhood in the definition relate to the use of the term “relative”?
2. Read Theorem 4.6 and its proof.
 - a. The proof is broken into two parts, one where $x_n < x_0$ and another part where $x_n > x_0$. Draw a picture of this situation and explain graphically why the first case allows you to conclude that $f'(x_0) \geq 0$ and why the second case allows you to conclude that $f'(x_0) \leq 0$.
 - b. Draw the corresponding picture for when f has a relative minimum at x_0 . Write out the complete proof for this case.
3. Read Theorem 4.7 (Rolle’s Theorem).
 - a. Draw a picture to illustrate the theorem. Make sure to label a , b , $f(a)$, $f(b)$, and c .
 - b. Why is it necessary for the derivative to exist on (a,b) ? Give an example where the derivative does not exist on this interval and the conclusion of Rolle’s Theorem does not hold.
 - c. Note that differentiability on (a,b) implies continuity on (a,b) . But Rolle’s Theorem also requires continuity at a and at b . Why is this necessary? Give an example where the function is not continuous at one of these endpoint, but is differentiable on the open interval (a,b) and the conclusion of Rolle’s Theorem does not hold.
 - d. Read the proof of Rolle’s Theorem. What other theorems does it invoke? How are they put together in this proof?
4. Read Theorem 4.8 (the Mean-Value Theorem).
 - a. Draw a picture to illustrate the theorem. Make sure to label a , b , $f(a)$, $f(b)$, and c .
 - b. Read the proof of the Mean-Value Theorem. Where is the graph of L in your diagram? What is the purpose of introducing L ?
 - c. What is the basic idea of this proof?
5. The Mean-Value Theorem is perhaps the most important theorem in the calculus (even more than the “Fundamental Theorem of Calculus” despite its lofty name). By tracing which theorems are used in its proof, which theorems were used in the proofs of those theorems, and so on, make a list of the big ideas needed to prove the Mean-Value Theorem.
6. Read Theorem 4.9 and its proof. For each of the parts i-iv, identify how the Mean-Value Theorem is used in the proof. Draw a graph to illustrate the proof.
7. (Extra)
 - a. Read Theorem 4.10. What key idea in introductory calculus is this theorem related to? Read the proof and note how the Mean-Value Theorem is used.
 - b. Read Theorem 4.12 and its proof, then read Theorem 4.13 and its proof. How is the Mean-Value Theorem used? Why is it important that $f(x_0) = g(x_0) = 0$ in the proof of L’Hospital’s Rule?