

**MAT 310**  
**Sections 1.3 – 1.4 Study Guide**

**Four-Point Geometry**

1. (Figure 1.3.1). The text explicitly states how the undefined terms “point” and “line” are represented in the models but does not say how the undefined term “on” is represented. State this explicitly.
2. (Four-Point Theorem 2).
  - a. What are the “simple combinatorics” referenced in the proof?
  - b. Explain in simpler terms why we know lines correspond one-to-one with pairs of points.
3. (Four-Point Theorem 3). The proof uses the technique of proof by contradiction. Explicitly say how it does this (i.e., What statement do we assume to be true? What contradiction do we arrive at? What can we then conclude?)
4. (Four-Point Theorem 4).
  - a. The second sentence of the proof asserts that two of the lines on  $P$  intersect  $l$ . How is this justified?
  - b. Why couldn't the assumed “second line parallel to  $l$ ” contain  $P$  without violating Four-Point Theorem 3?
  - c. State explicitly how proof by contradiction is used in part of this proof.

**Fano's Geometry**

5. (Figure 1.3.3).
  - a. How are “points,” “lines,” and “on” represented in this model?
  - b. What does it mean for an axiomatic system to be categorical? Do you think this system is categorical?
  - c. Construct an isomorphism between Figures 1.3.3 and 1.3.4.
6. (Fano's Theorem 1). You've seen this proof a few times now. Where?
7. (Fano's Theorem 2).
  - a. In the second sentence of the proof, why must the three lines be distinct?
  - b. In the third sentence the phrase “these three points” is vague. Actually there is no antecedent in what is written, so maybe some sentence got left out. What three points is this sentence talking about?
  - c. Draw a picture including the 7 points and the lines discussed in the proof through the third sentence. Use your picture to help explain why the assertion in the third sentence is correct.
  - d. Continue adding to your diagram and use it to help understand how proof by contradiction is used to show that there are exactly 7 points.
  - e. To prove that there are 7 lines, how will you have to modify the proof of Four-Point Theorem 2?

## Young's Geometry

8. (Figure 1.3.5). Draw a model like that in Figure 1.3.3 for Young's Geometry.
9. (Young's Theorem 1). Sketch a figure to illustrate the proof.
10. (Young's Lemma 1).
  - a. Sketch a diagram of the statement of the theorem.
  - b. Sketch a diagram of the statement assumed in the first sentence of the proof. Show how the following statements in the proof are now illustrated in your diagram.
11. (Young's Theorem 2).
  - a. Explain why  $R$  is not on  $l$ .
  - b. Explain why lines  $m$  and  $n$  must be distinct.
  - c. Sketch a diagram of the statement assumed in the first sentence of paragraph 2. Show how the following statements in the proof are now illustrated in your diagram.

## Incidence Geometry

12. (Incidence Axioms).
  - a. Read Incidence Axioms 1 & 2. Without glancing ahead, what do you think Theorem 1 will be?
  - b. Verify that the geometries in Examples 1.4.2 and 1.4.3 satisfy the Incidence Axioms.
13. (Incidence Theorems 1-3). Identify previous theorems for which you have already proven almost these exact same statements.
14. (Parallel Axioms). Illustrate the three possibilities for parallel axioms in Incidence Geometry.