Hyperbolic Banach-Tarski Paradox

Consider the subgroup of isometries of the Poincare disk generated by $\{t, s\}$ where

t is clockwise rotation by 120° about the origin

s is rotation is rotation by 180° about a second point as shown in the diagram.

We will decompose the Poincare disk into three congruent subsets with the property that

$t(A) = B$, $t(B) = C$, $t(C) = A$, and

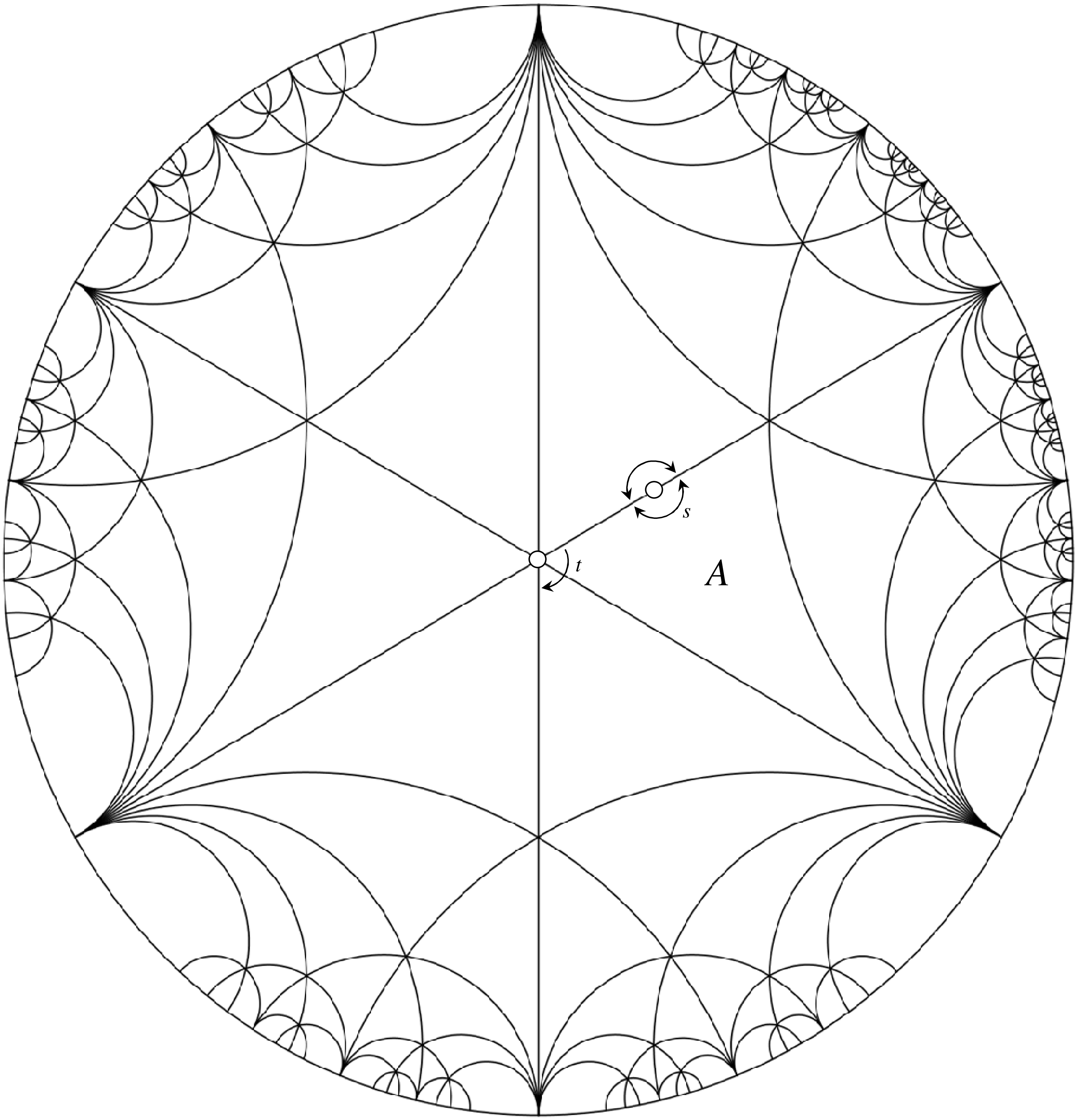
$s(A) = B \cup C$, $s(B \cup C) = A$

This implies that *A* is simultaneously 1/3 and 1/2 of the Poincare disk.

To generate the decomposition, iteratively apply the conditions for *t* and *s* listed above to the triangle* initially labeled as belonging to set *A*. Choose to place triangles in the image of *A* under *s* into set *B* when you have the choice.

* Note that the decomposition is actually formed out of 1/3-ideal triangles.

This activity is based off of the article:
Wagon, S., A Hyperbolic Interpretation of the Banach-Tarski paradox, *The Mathematica Journal, 3* (4), 1993, 58-61.



s
t
A