

**MAT 310**  
**Homework 13**

1. Use spherical coordinates for a sphere of radius  $r$  for the following questions.
  - a. Parameterize a path  $\gamma: [0,1] \rightarrow M$  for a circle at latitude  $\varphi_0$ .
  - b. Find the velocity  $\gamma'(t)$  and acceleration  $\gamma''(t)$  vectors for this path in  $\mathbf{R}^3$ . Explain why the velocity vector is intrinsic but the acceleration vector is extrinsic.
  - c. Find the covariant derivative  $\nabla_{\gamma'(t)}\gamma'(t)$  by projecting the acceleration vector  $\gamma''(t)$  onto the tangent plane  $T_{\gamma(t)}M$ .
  - d. Show that the covariant derivative  $\nabla_{\gamma'(t)}\gamma'(t)$  is only zero at the poles and the equator. Explain what this means about geodesics on the sphere.

2. Consider the following coordinates for a cylinder of radius  $r$

$$x=r\cos\theta$$

$$y=r\sin\theta$$

$$z=h$$

- a. Describe what all paths look like on the cylinder parameterized by

$$x=r\cos(at)$$

$$y=r\sin(at)$$

$$z=bt+c$$

for different values of  $a$ ,  $b$ , and  $c$ .

- b. Show that these paths are all geodesics on the cylinder by showing that the covariant derivative is zero.
3. The latitude of Tempe, AZ is  $33^\circ 23'$  N. Determine the scalar speed ( $s$ ) and the scalar intrinsic acceleration ( $a$ ) for the path at this latitude. Recall that the rate at which  $v$  is turning is  $a/s$ . Determine this rate and multiply by the duration of one revolution to find the total amount rotation of the velocity vector.
  4. Visit the Foucault pendulum in PSF twice and record the location of the pendulum's swing. Use your data points to determine how much the pendulum rotates in exactly 24 hours.