

Hilbert's Axioms:

I. Axioms of incidence

Postulate I.1. For any two points there exists a line that contains each of the points.

Postulate I.2. For any two points there exists no more than one line containing both.

Postulate I.3. There exist at least two points on any given line. There exist at least three points that do not lie on a given line.

Postulate I.4. For a set of three points that do not lie on the same line, there exists a plane that contains each of the points in the set. For every plane there exists at least one point which it contains.

Postulate I.5. For a set of three points that do not all lie on the same line, there exists only one plane that contains each of the points in the set.

Postulate I.6. If two points of a line, a , lie in a plane, α , then every point in a lies in α .

Postulate I.7. If two planes have a point in common, then they have at least one other point in common.

Postulate I.8. There exists at least four points which do not lie in a plane.

II. Axioms of order

Postulate II.1. If a point B lies between points A and C , then the points A , B , and C are three distinct points on the same line and B lies between C and A .

Postulate II.2. Given two points A and C , a point B exists on the line \overline{AC} such that C lies between A and B .

Postulate II.3. Given any three points of a line, one and only one of the points is between the other two.

Postulate II.4. Given three points A , B , and C that do not lie on a line and given a line, a , that lies in the plane containing A , B , and C but does not contain any of the points A , B , or C : if the line a passes through a point of the segment \overline{AB} , then it also passes through a point in the segment \overline{AC} or through a point in the segment \overline{BC} .

III. Axioms of congruence

Postulate III.1. Given two points A and B on a line a and given a point A' on a or another line a' , there exists a point B' on a side of the line a' such that \overline{AB} and $\overline{A'B'}$ are congruent.

Postulate III.2. Given segments $\overline{A'B'}$ and $\overline{A''B''}$ such that both are congruent to the same segment \overline{AB} , then $\overline{A'B'}$ is congruent to $\overline{A''B''}$.

Postulate III.3. Given a line a with segments \overline{AB} and \overline{BC} such that the point B is the only intersection of the two points and on the same line or a line a' with segments $\overline{A'B'}$ and $\overline{B'C'}$ such that the point B' is the only intersection: if \overline{AB} is congruent to $\overline{A'B'}$ and \overline{BC} is congruent to $\overline{B'C'}$ then \overline{AC} is congruent to $\overline{A'C'}$.

Postulate III.4. If $\angle ABC$ is an angle and $\overline{B'C'}$ is a ray, then there is one and only one ray $\overline{B'A'}$ on each side of the line $\overline{B'C'}$ such that $\angle A'B'C'$ is congruent to $\angle ABC$.

Corollary: Every angle is congruent to itself

Postulate III.5. Given two triangles $\triangle ABC$ and $\triangle A'B'C'$ with \overline{AB} congruent to $\overline{A'B'}$, \overline{AC} congruent to $\overline{A'C'}$ and $\angle BAC$ congruent to $\angle B'A'C'$ then $\triangle ABC$ is congruent to $\triangle A'B'C'$.

IV. Axiom of parallels

Postulate IV.1. Given a line a and a point A not on a , there is at most one line in the plane that passes through A and does not intersect a .

V. Axioms of continuity

Postulate V.1 (Archimedes axiom). Given segments \overline{AB} and \overline{CD} , there exists a number n such that n copies of \overline{CD} constructed contiguously from A along the ray \overline{AB} will pass beyond the point B .

Postulate V.2 (line completeness). There exists no extension of a set of points on a line with order and congruence relations that would preserve the relations existing among the original elements as well as preserving line order and congruence, i.e., Axioms I-III and V.1.