Mystery Constant P

Our approximation is to determine the probability of **winning** at Craps. The probability range is from 0 (lose) to 1 (win). The game is played with two six sided die summed together. The object is to role a 7 or 11 on the first roll or roll the same number (no matter the combination) on a subsequent roll without rolling a 7.

With 36 potential combinations of rolls there are 8 potential winning combinations for the first roll. This makes our odds of winning on the first

	1	2	3	4	5	6
1	L	-	-	-	-	W
2	-	-	-	-	W	-
3	-	-	-	W	-	-
4	-	-	W	-	-	-
5	-	W	-	-	-	W
6	W	-	-	-	W	-

roll 8 out of 36 or 8/36. This does factor to $2/9^{ths}$ probability of winning. If you do not win on the first roll, you must roll again until you either win or lose.

The function to determine the probability of wining at craps is defined as P where:

$$P={}^{2}/_{9} + {}^{1}/_{72}\left[\mathbf{1}+{}^{3}/_{4}+\left({}^{3}/_{4}\right)^{2}+\left({}^{3}/_{4}\right)^{3}+\left({}^{3}/_{4}\right)^{4}....\right]$$
$$+ {}^{1}/_{81}\left[\mathbf{1}+{}^{13}/_{18}+\left({}^{13}/_{18}\right)^{2}+\left({}^{13}/_{18}\right)^{3}+\left({}^{13}/_{18}\right)^{4}....\right]$$
$$+ {}^{25}/_{648}\left[\mathbf{1}+{}^{25}/_{36}+\left({}^{25}/_{36}\right)^{2}+\left({}^{25}/_{36}\right)^{3}+\left({}^{25}/_{36}\right)^{4}....\right]$$

*Note – We are unable to illustrate the origin of the formula presented for subsequent rolls.

Not all parts of the formula will be used. The **first** roll is simply $P={}^{2}/_{9}$. The **second** roll is:

$$P={}^{2}/_{9} + {}^{1}/_{72}[\mathbf{1}] + {}^{1}/_{81}[\mathbf{1}] + {}^{25}/_{648}[\mathbf{1}]$$

third roll is

$$P={}^{2}/_{9} + {}^{1}/_{72}\left[\mathbf{1}+{}^{3}/_{4}\right] + {}^{1}/_{81}\left[\mathbf{1}+{}^{13}/_{18}\right] + {}^{25}/_{648}\left[\mathbf{1}+{}^{25}/_{36}\right]$$

Fourth roll is:

$$P={}^{2}/_{9} + {}^{1}/_{72}\left[\mathbf{1}+{}^{3}/_{4}+\left({}^{3}/_{4}\right)^{2}+\left({}^{3}/_{4}\right)^{3}\right]$$
$$+ {}^{1}/_{81}\left[\mathbf{1}+{}^{13}/_{18}+\left({}^{13}/_{18}\right)^{2}+\left({}^{13}/_{18}\right)^{3}\right]$$
$$+ {}^{25}/_{648}\left[\mathbf{1}+{}^{25}/_{36}+\left({}^{25}/_{36}\right)^{2}+\left({}^{25}/_{36}\right)^{3}\right]$$

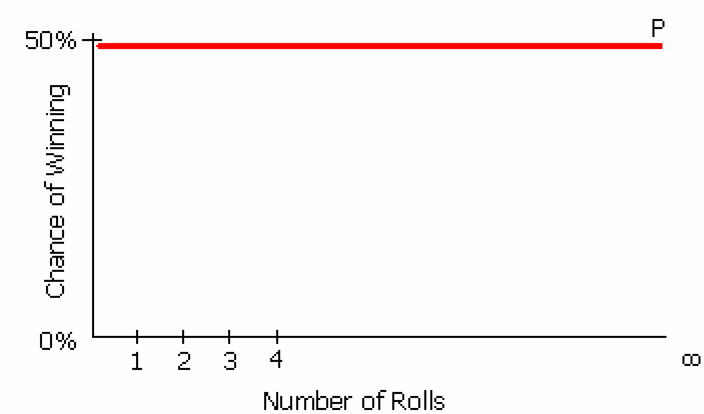
Hopefully, you see the pattern emerging. With each subsequent roll, additional terms are added to the equation, increasing the odds of winning.

The Unknown

The unknown is represented as P. P is the percentage chance of winning at craps within a specific number of rolls not limited to an infinite amount of rolls. We define P as:

$$P = {}^{2}/_{9} + {}^{1}/_{72}[1 + {}^{3}/_{4} + ({}^{3}/_{4})^{2} + ({}^{3}/_{4})^{3} + ({}^{3}/_{4})^{4} \ldots.]$$
$$+ {}^{1}/_{81}[1 + {}^{13}/_{18} + ({}^{13}/_{18})^{2} + ({}^{13}/_{18})^{3} + ({}^{13}/_{18})^{4} \ldots.]$$
$$+ {}^{25}/_{648}[1 + {}^{25}/_{36} + ({}^{25}/_{36})^{2} + ({}^{25}/_{36})^{3} + ({}^{25}/_{36})^{4} \ldots.]$$

And the results would look like:



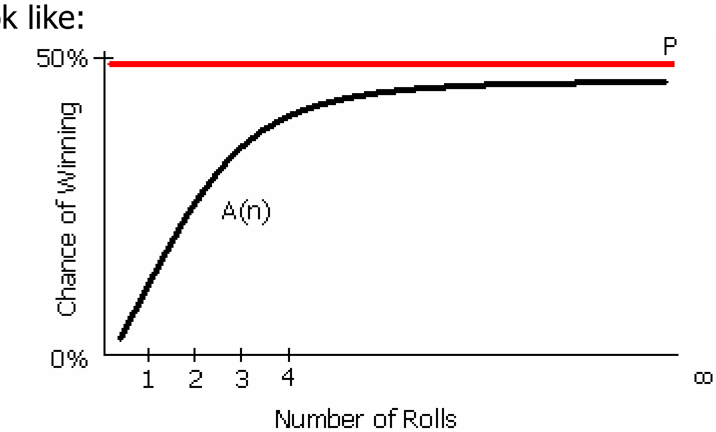
Approximations

Our approximations, or A(n), indicate the approximate percentage chance of winning at craps given a specific roll (with error). Previously, we indicated the way to use the pre-determined formula appropriately to coincide with the amount of rolls.

Rather than re-writing the formula out here is a simplified version for the Approximations given that n is the current number of rolls (potentially infinite):

$${}^{2}/_{9} + {}^{1}/_{72}\sum_{K=0}^{n}({}^{3}/_{4})^{k} + {}^{1}/_{81}\sum_{K=0}^{n}({}^{13}/_{18})^{k} + {}^{25}/_{648}\sum_{K=0}^{n}({}^{25}/_{36})^{k}$$

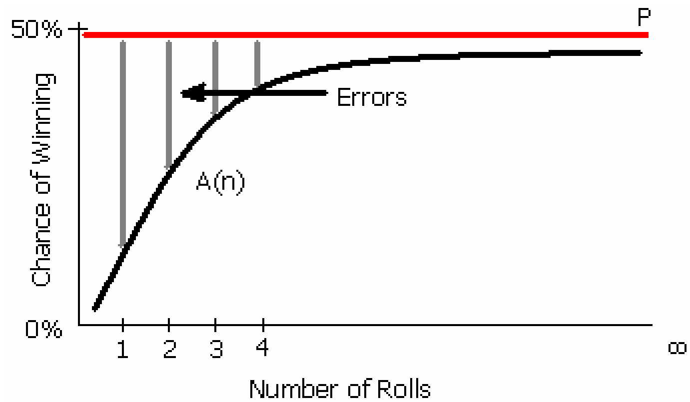
This would look like:



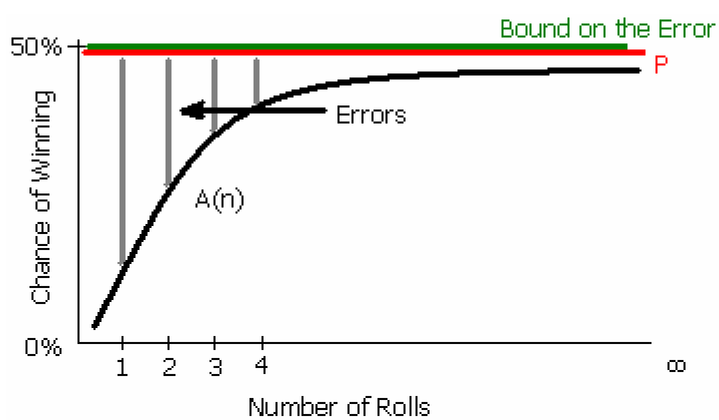
Error
The error, simply noted as "error", is the different between P and the current approximation, or the difference between our calculated chance of winning and the actual chance of winning. Unfortunately, we don't know the true value of P; therefore, the error is an approximation also.

You can determine this by: error=|P-A(n) |
*Note: Either of the formulas above can be substituted for A(n)

This would look like:



Bound on the Error
You would think that with a potential of infinite rolls the odds would become either win or lose, rather than roll again. For this reason, we chose to set our upper limits to 50%. The bound on the error is represented by an ε (epsilon) symbol. Mathematically, we can represent ε as ε=|.50-A(n) |>error, also as ε>error. Contextually, the bound on the error is will always be greater than true error; consequently, that would mean that .50>P. A graph of what the bound on the error would look like is below:



To help you understand this better we have included one of many possibilities of m-file code below. We hope it is helpful.

```
clear all
format long

x=3/4;
y=13/18;
z=(25/36);

n=6;
A(1)=2/9;


for k=2:n
    A(k)=A(k-1)+((1/72)*(x^(k-2)))+((2/81)*(y^(k-2)))+((25/648)*(z^(k-
2)))
end

x=1:n;
plot (x,A(x),'*:')
```