MATLAB Project 4 MAT 294
Orbital Mechanics Due 12-5-05

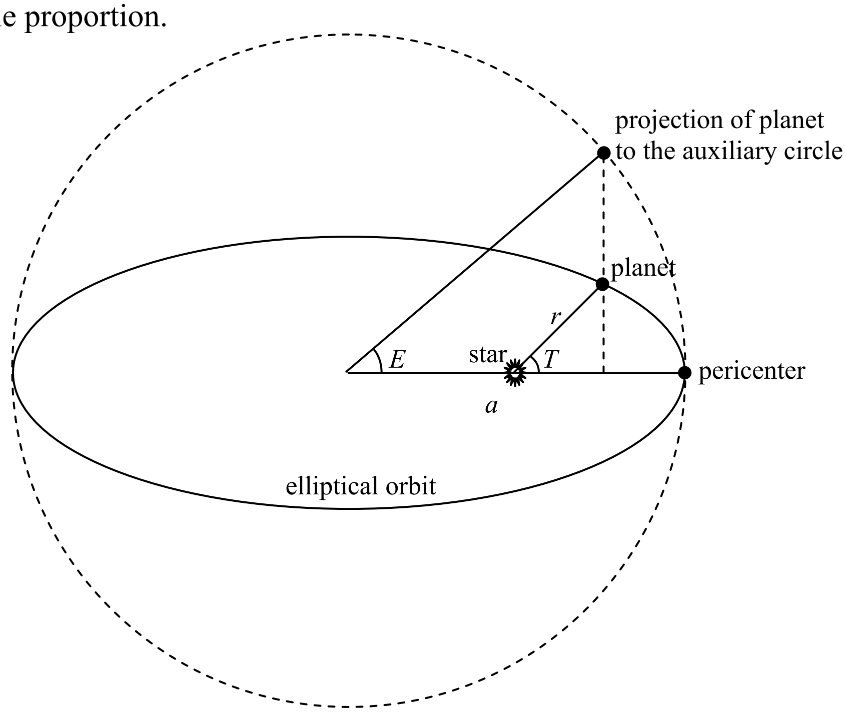
Kepler's first law of planetary motion states that the orbit of a planet is an ellipse with the star at one focus. His second law states that the line joining the planet and the star sweeps out equal areas during equal intervals of time. If we know the period (length of time for one complete orbit), the shape of the ellipse, and the time of the pericenter (when the planet is closest to the star), then Kepler's first and second laws are sufficient to determine the location of the planet in orbit at any other time. You will write a MATLAB program to do this.

There are three important angles about an orbit that you will need:

The *true anomaly* (T) is the angle measured at the star between the pericenter and the planet. This gives the planet's position relative to the star, so it is the angle we will be trying to determine.

The *eccentric anomaly* (E) is the angle measured at the center of the ellipse from the pericenter to the projection of the planet on the auxiliary circle. This is an intermediate angle that we will need in our computations.

The *mean anomaly* (M) is the angle from the pericenter that would have been swept out by the planet if it were moving at a constant angular velocity. This is a useful angle because it changes at a constant rate in time and can easily be converted into time units by a simple proportion.



Finally, the shape of the ellipse is measured by its *eccentricity* (e) which is the fraction of the distance along the semimajor axis at which the focus lies. The distance from the star to the planet (r) can then be determined from

$$r = a\left(1 - e\cos E\right) \qquad (0)$$

Where a is the length of the semimajor axis.

Basic geometry allows one to determine the following relations among the angles defined on the previous page:

$$M = E - e\sin E \qquad (1)$$

$$\tan\frac{T}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \qquad (2)$$

The general strategy to determine the position of a planet at any time is to determine *M* based on the fraction of one orbit that has elapsed. Use the first equation above to determine *E*. Then use the second equation above to determine *T*. The only difficulty is that the first equation cannot be explicitly solved for *E*. Thus your program will have to apply Newton's method to this equation.

Your program must give the true anomaly, *T*, and distance from the star, *r*, at regular time intervals for an entire orbit. The program should start by defining the following variables so that they can later be changed by the user:

- The semimajor axis of the orbit
- The eccentricity of the orbit
- The time required for one orbit (period)
- The time change between each calculation to be made

Your program should be structured as a loop that runs through an entire orbit at the specified time intervals. The easiest way to do this is to use a "for" loop. See us if you do not remember the syntax. Inside this loop, you should compute, *M*, then iterate Newton's method 3 times with equation (1) to get *E*. This is also best done with a "for" loop. Then, still inside the loop, use equation (2) to determine *T*. You can use equation (0) to determine *r*. Then you can give the values for *T* and *r* on the same line with the simple statement

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disp([T,r])
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You can use the following information to run your program for both Earth and Mars using increments of one day on that planet:

Planet	Eccentricity	Semimajor Axis (Astronomical Units)	Heliocentric Period (terrestrial days)	Equatorial Period (hours)
Earth	0.0167175	1.000	365.256	23.9345
Mars	0.0933865	1.489	686.980	24.6229

Copy and paste your output for each of these runs into an email, attach your m-file program and email the whole set to vicki@mathpost.asu.edu.

Remember that the first line of the program should be a comment with your name. The second line should be the comment "% Orbital Mechanics" to indicate the assignment.