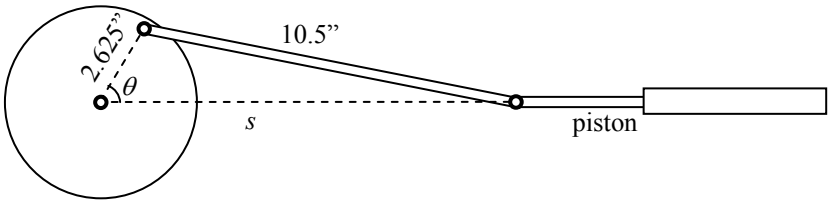
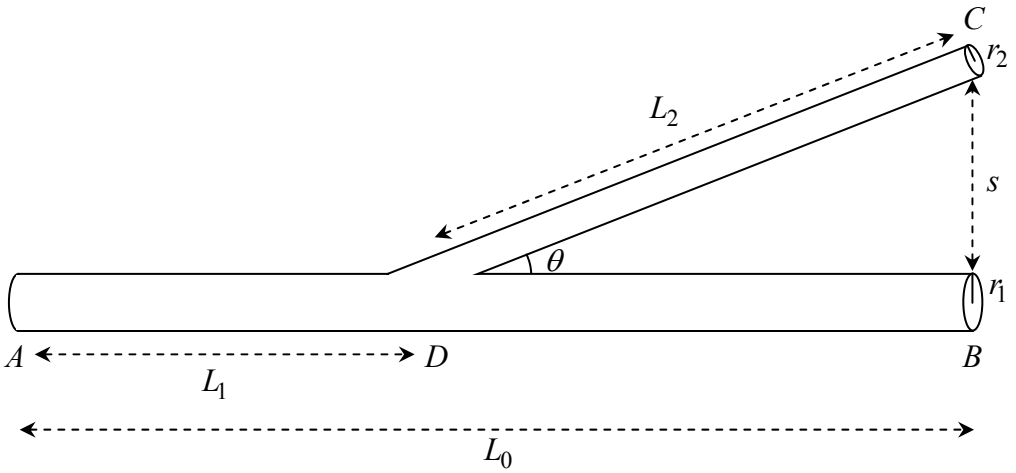
Engine Velocity: The diagram below gives the basic dimensions of the piston and crankshaft in the 1937 John Deere B engine:

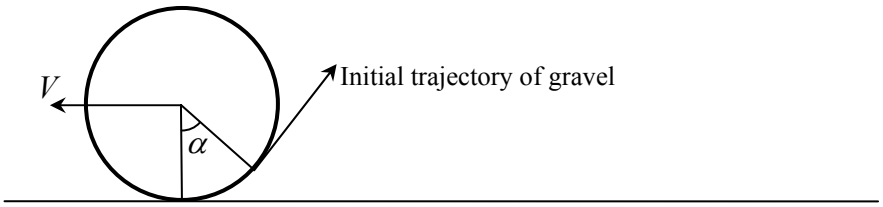


Given that the flywheel rotates at a speed of 1,340 revolutions per minute, find the maximum velocity of the piston (in miles per hour).

Blood Vessel System: The body's system of blood vessels is made up of arteries, arterioles, capillaries, and veins. The transport of blood from the heart through all organs of the body and back to the heart should be as efficient as possible. One way this can be done is by having large enough blood vessels to avoid turbulence, with blood cells small enough to minimize viscosity. You will find the angle θ for branches in blood vessels such that total resistance to the flow of blood is minimized. Assume that a main vessel of radius r_1 runs along the horizontal line from *A* to *B*. A side artery, of radius r_2, heads for a point *C*. Choose point *B* so that *CB* is perpendicular to *AB*. Let *CB* = *s* and let *D* be the point where the axis of the branching vessel cuts the axis of the main vessel. According to Poiseuille's law, the resistance *R* in the system is proportional to the length *L* of the vessel and inversely proportional to the fourth power of the radius *r*. That is, $R = k \cdot \frac{L}{r^4}$, where *k* is a constant determined by the viscosity of the blood. Let $AB = L_0$, $AD = L_1$, and $DC = L_2$. The total resistance from *A* to *C* is the sum of the resistance on *AD* and the resistance on *DC*. Find the angle θ that minimizes this resistance.



Flying Gravel: The grooves or tread in a tire occasionally pick up small pieces of gravel, which then are often thrown into the air as they work loose from the tire. When following behind a vehicle on a highway with loose gravel, it is possible to determine a safe distance to travel behind the vehicle so that your automobile is not hit with flying debris. For different angles, α, at which the gravel is thrown from the tire, the distance traveled by the gravel will vary. Find the maximum distance that the debris could be thrown given a velocity V.



Maximum Sustainable Harvest: Standard population models allow us to predict the population of a species one year depending on the previous year's population. One such model determines the population of salmon next year by the function $f(S) = Se^{r(1-S/P)}$, where S is this year's salmon population, P is the natural equilibrium population, and r is a constant that depends upon how fast the population grows. For different population sizes S, different amounts of salmon can be fished (harvested) in a year so that the population remains the same size. Such a harvest is sustainable over time. Find the maximum sustainable harvest. Do this by first finding the population S_0 which will support the maximum sustainable harvest (this will depend on r and P). Then find the size of such a harvest (which will depend on S_0, r, and P).

Bird Migration: The amount of time a migrating bird can fly depends on its velocity, that is it is some function $T(v)$. If E is the bird's initial energy, then the bird's effective power is given by kE/T, where k is the fraction of the power that can be converted into mechanical energy. According to principles of aerodynamics, $\frac{kE}{T} = aSv^3 + I$, where a is a positive constant, S is the wind speed, and I is the induced power, or rate of working against gravity. Find the velocity that a bird would need to fly to migrate a maximum distance (this will depend on some, but not all, of the parameters listed above).