Extra Credit – 20 points
Due: Monday, October 17

Constrained growth of a population is often modeled by what we call the "logistic model." Applied to a biomass of yeast cells, for example, we may write the logistic function

$$M(t) = \frac{K}{1 - \left(\frac{M_0 - K}{M_0}\right)e^{-rt}}.$$

where
$M(t)$ is the mass of the collection of yeast cells (in grams) after t hours,
K is the carrying capacity (maximum mass) of the environment containing the cells,
M_0 is the initial mass (at time t=0), and
r is the maximum growth rate as a proportion of the population.

Suppose that an initial mass of 9.6 grams (g) is incubated in a Petri dish, and it is determined (by fitting a curve to the data) that r=.625 and K=587 g.

1. Determine the average rate of change of M with respect to t from t=2 hours to t=4 hours. Explain what this means in the context of the yeast cells.

2. Use your Matlab program to approximate the instantaneous rate of change of M with respect to t at t=2 hours to an accuracy of 0.1 grams per hour. Explain what this means in terms of the yeast cells.

3. Find a place where the growth of the yeast cells is slowing down. You may have to graph the function to do this. Repeat question #2 for this point in time.

4. Answer the five approximation questions symbolically, numerically, graphically, and in the context of the yeast cells.