MAT 294

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Sealey/Oehrtman

Approximation Framework

Elements:

- An unknown value
- Approximations (or estimates)
- Errors: each approximation is associated with an error, i.e.,

  Error = | Unknown – Approximation |.
- Bounds on the error: an approximation is useless if you do not know a bound on the error, i.e.,

  Error < ε

  or

  | Unknown – Approximation | < ε

Logic:

- A bound on the error allows you to use an approximation to restrict the range of possibilities for the unknown value:

  Approximation – ε < Unknown < Approximation + ε .
- Accuracy: the error can be made small.
- Precision: the difference between the approximations can be made small.
- Practicality: for any bound, you want to find an approximation with an error smaller than that bound (the error can be made as small as you wish).

Limit Notation

$$\lim_{cv \to s} A = U$$

U = Unknown (what is being approximated)
A = Approximation
cv = controlling variable
s = singularity point for cv (typically the unknown cannot be computed directly)

Questions to ask in an approximation situation:

1. What am I trying to approximate (the unknown)? U
2. What are the approximations? A
3. What are the errors? $|U - A|$
4. How can I bound the errors? $|U - A| < \varepsilon$
5. How can I make the bound on error small? $cv \to s$

You should be able to answer these questions symbolically, graphically, numerically, and in context.