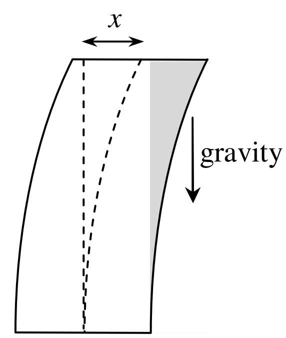
Lab 5

Exploration of Nonlinear Systems

Consider the swaying motion of the top of a skyscraper. Measure the horizontal displacement x of the top of the building so that $x = 0$ when the skyscraper is perfectly vertical. Then assume that the building acts somewhat like a spring with a restoring force proportional to the displacement pulling it back toward its vertical position and a damping force, also proportional to its displacement. To complete the simple model, we must account for the fact that when the skyscraper sways, a portion of the building is not directly above the foundation. Thus the force of gravity will cause the building to buckle further. This is called a P-Delta effect, and for small displacements, it is much smaller than the restorative force but becomes more significant for larger displacements. A simple model of this effect is equivalent to including a horizontal force at the top of the building proportional to x^3.



1. Assume all constants of proportionality are 1, and write the second order ODE modeling the displacement x for the swaying skyscraper. Rewrite this as a system of first-order equations. Use MATLAB to plot trajectories in a phase plane for two different nontrivial initial conditions of your choosing. Explain what your results mean in terms of the behavior of the skyscraper.

2. If you added the two solutions from Question 1, would this also be a solution to the system of equations. Add the components of your two sets of initial conditions and use this as a new initial condition and plot the solution using MATLAB to verify your response. Explain why this is happening.

3. Find the equilibrium solutions. Find the corresponding linear equations at each of these points and use MATLAB to find the eigenvectors and eigenvalues for each linear system. Use this information to draw a phase plane with a representation of the local behavior near each equilibrium point. Include arrows to show the direction of flow for the trajectories. Explain what each equilibrium point and the local behavior means in terms of the swaying skyscraper.

4. The local straight line solutions that enter each equilibrium point are especially important because if you follow them backwards in time they determine a region of initial conditions in the phase plane for which the differential equations predict the building will not fall down. These are lines called separatrix. For each separatrix solution, use the eigenvalue and eigenvector information from the previous problem to determine an initial condition on that solution but very near the equilibrium point. If you have chosen a point close enough, the numeric solution should essentially produce the straight-line solution going into the

equilibrium point. Verify that this happens for at least one of your choices. Now you want to find the solution backwards in time to see where the full separatrix lies. Use ode45 with initial time 0 and final time -10 to do this. You may use the command “hold on” to make MATLAB plot the results on a single graph. Provide a sketch of the results and shade in the region of initial conditions for which the building will fall down.

5. Write the nonlinear second order ODE for a 2 meter long simple pendulum (see Lab 3) and write the corresponding system of first order equations. Find and characterize the equilibrium points. Which ones are stable and which ones are unstable? Which ones are asymptotically stable? Draw the separatrix on a phase plane and indicate what type of behavior occurs for initial conditions on each region in the diagram.

6. Earlier in the semester, we studied systems of rate of change equations designed to inform us about the future populations for two species that are either competitive (that is both species are harmed by interaction) or cooperative (that is both species benefit from interaction). Which of the following systems describes a situation where the two species are competitive? Explain why.

$$\frac{dx}{dt} = -5x + 2xy \qquad \frac{dx}{dt} = 3x\left(1 - \frac{x}{3}\right) - \frac{1}{10}xy$$

$$\frac{dy}{dt} = -4y + 3xy \qquad \frac{dy}{dt} = 2y\left(1 - \frac{y}{10}\right) - \frac{1}{5}xy$$

Find the equilibrium solutions for the system modeling the populations of the competitive species. Determine the corresponding linear system of differential equations about each equilibrium solution and use the information you gain about the solutions near each of these points to sketch the phase portrait.

7. Without using technology, use the tools of linearization and nullclines to sketch the phase portrait for the nonlinear system

$$\frac{dx}{dt} = \cos(y)$$

$$\frac{dy}{dt} = y - x$$

Be as accurate as possible and show all supporting work.