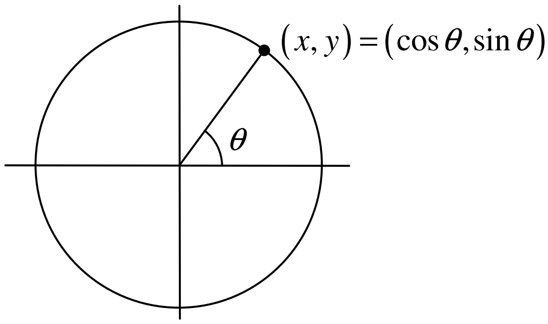
Trigonometric Functions:

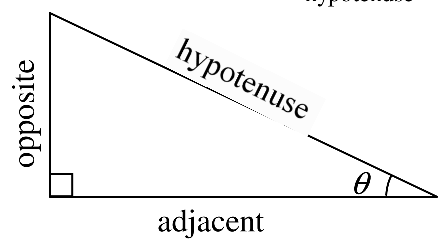
Unit Circle Definition



Right Triangle Definition

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

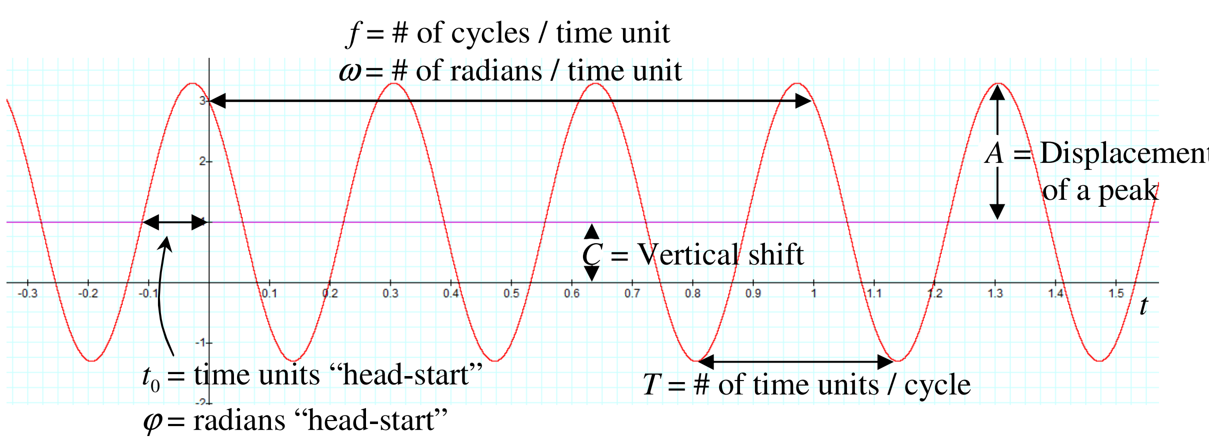
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



• The *radian* is a unit of angular measure defined such that an angle of one radian at the center of a circle produces an arc length equal to the radius of the circle. Since a circle of radius *r* has a circumference of $2\pi r$, there are 2π radians in a full circle and $\pi/2$ radians in a right angle. • $\cos\theta$ is the *x*-coordinate of the point on the unit circle at θ radians counterclockwise from the positive *x*-axis. • $\sin\theta$ is the *y*-coordinate of the point on the unit circle at θ radians counterclockwise from the positive *x*-axis.	• The *hypotenuse* of a right triangle is the triangle's longest side, i.e., the side opposite the right angle. • Given one of the two other angles, the *opposite* is the side that does not end at that angle. The *adjacent* is the side extending from the angle to the right given angle. • $\cos\theta$ is the ratio of the lengths of the adjacent and hypotenuse. • $\sin\theta$ is the ratio of the lengths of the opposite and hypotenuse.

Primary forms for modeling periodic behavior	**Parameters**
$y = A\sin(\omega t + \varphi) + C$	A: amplitude ω: angular frequency φ: phase shift C: vertical shift
$y = A\sin(2\pi f t + \varphi) + C$	A: amplitude f: frequency φ: phase shift C: vertical shift

Other forms for modeling periodic behavior	**Parameters**
$y = A\sin\left(\frac{2\pi}{T}t + \varphi\right) + C$ $y = A\sin\left(\frac{2\pi}{\lambda}x + \varphi\right) + C$	A: amplitude T: period λ: wavelength φ: phase shift C: vertical shift
$y = A\sin(\omega(t + t_0)) + C$ $y = A\sin(\omega(x + x_0)) + C$	A: amplitude ω: angular frequency t_0: time shift x_0: position shift C: vertical shift



Key relationships between the parameters:

$\omega = 2\pi f$

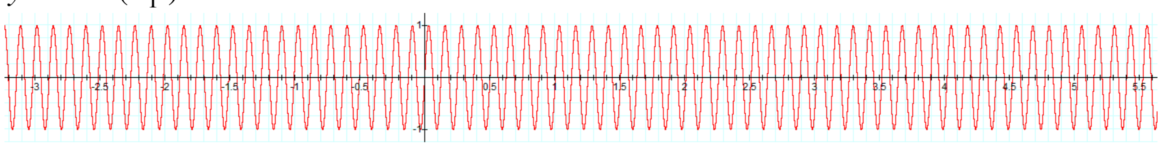
$\varphi = \omega t_0$

$f = 1/T$

Superposition & Beats:

$y = A\sin(\omega_1 t) + A\sin(\omega_2 t)$ $= 2A\sin\left(\frac{\omega_1+\omega_2}{2}t\right)\sin\left(\frac{\omega_1-\omega_2}{2}t\right)$	A: input amplitudes $2A$: modulated amplitude ω_1, ω_2 : input angular frequencies $\frac{\omega_1+\omega_2}{2}$: oscillation angular frequency $\frac{\omega_1-\omega_2}{2}$: modulation angular frequency

$y = A\sin(\omega_1 t)$



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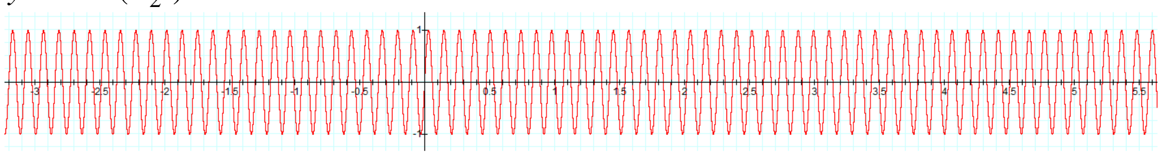
$y = A\sin(\omega_2 t)$



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$y = 2A\sin\left(\frac{\omega_1+\omega_2}{2}t\right)\sin\left(\frac{\omega_1-\omega_2}{2}t\right)$