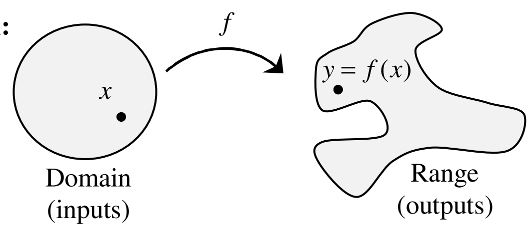
Functions

Notation: $f(x) = y$ means that the function f maps the input x to the output y.

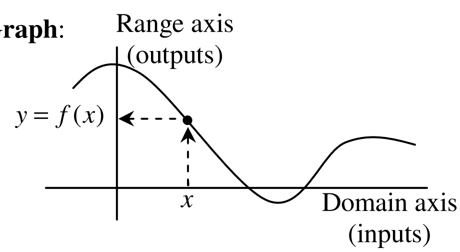
Terminology:

- The *domain* is the set of inputs.
- The *range* is the set of outputs.
- Each input must produce a *single output*.
- Different inputs may produce the same output. For example $f(x) = x^2$ has $f(-2) = f(2) = 4$, so the output 4 is produced by inputs of –2 and 2.
- If each input produces a different output, we say the function is *one-to-one*. For example, $f(x) = 3x - 5$ produces a different output for every input.
- If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ on some interval, we say that f is *increasing* on that interval.
- If $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ on some interval, we say that f is *decreasing* on that interval.

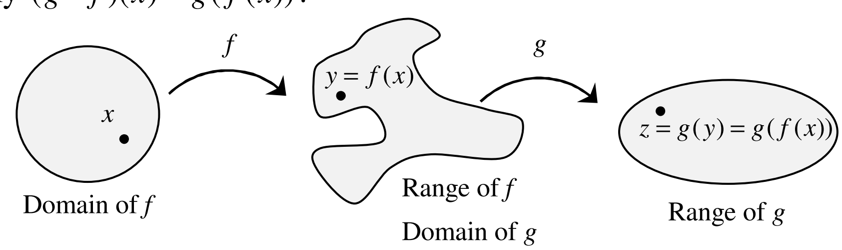
Diagram:



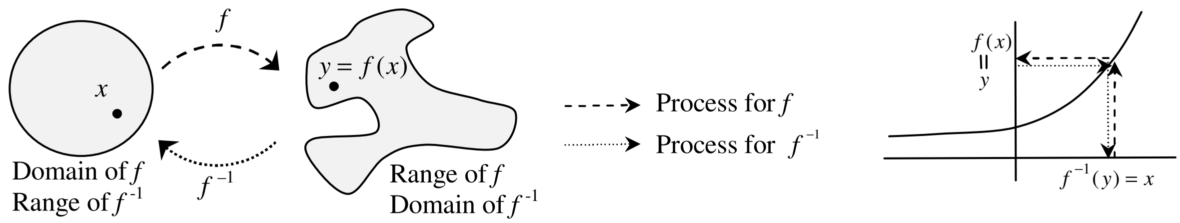
Graph:



Composition: If the range of one function, f, is within the domain of a second function g then we can compose them to get a new function $g \circ f$ by using the output of f as the input for g. Specifically $(g \circ f)(x) = g(f(x))$.



Inverse: If $f : X \to Y$ is one-to-one, then there is an inverse function $f^{-1} : Y \to X$ that undoes the action of *X*. That is, if $f(x) = y$ then $f^{-1}(y) = x$



Warning: The inverse notation looks a lot like an exponential power of -1. However, $f^{-1}(x)$ and $f(x)^{-1}$ are two very different things!

$$f^{-1}(x) \text{ means function inverse}$$

$$f(x)^{-1} \text{ means reciprocal or } \frac{1}{f(x)}$$

Note: If you can explain why $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (and not $g^{-1} \circ f^{-1}$) then you probably understand the ideas of composition and inverse pretty well.