Arithmetic

Types of numbers

Natural: $\{1, 2, 3, 4, \ldots\}$

Integer: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

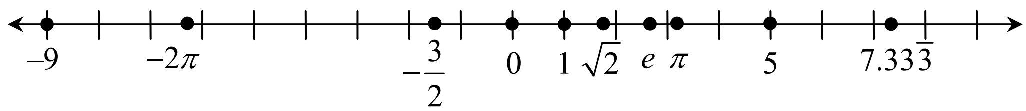
Rational: Everything of the form $\frac{p}{q}$ where p and q are integers
Everything that can be represented by a repeating or terminating decimal

Real: Everything that can be represented by an infinite decimal

Irrational: Real numbers without the rationals
Everything that is represented by an infinite non-repeating decimal

Number line, ordering and interval notation

Every real number is represented as a point on a continuous line.
Although points must be drawn with some size, they are imagined to be infinitely small.



Between every two rational numbers are infinitely many irrationals.
Between every two irrational numbers are infinitely many rationals.

For any two real numbers x an y, exactly one of the following must be true:

$x < y$, $x = y$, or $x > y$.

If $x < y$, then x is drawn to the left of y on the number line.

We use parentheses and brackets to denote intervals:

(a,b) = all real numbers x such that $a < x < b$

$[a,b)$ = all real numbers x such that $a \le x < b$

$(a,b]$ = all real numbers x such that $a < x \le b$

$[a,b]$ = all real numbers x such that $a \le x \le b$

Although infinity is not a number, we use it in interval notation as follows:

$(-\infty,b)$ = all real numbers x such that $x < b$

$(-\infty,b]$ = all real numbers x such that $x \le b$

(a,∞) = all real numbers x such that $a < x$

$[a,\infty)$ = all real numbers x such that $a \le x$

$(-\infty,\infty)$ = all real numbers

A common interpretation of multiplication and division

Suppose x and y are any real numbers and n is a natural number.

- Multiplication: $x = n \bullet y$ is a quantity created from n pieces, each of size y.
  That is, multiplication can be viewed as repeated addition:
  $$n \bullet y = \underbrace{y + y + \cdots + y}_{n \text{ times}}$$
- Division reverses this process
  - Equal partitioning model: We can think of dividing x into n pieces:
    $\frac{x}{n} = y$ is the quantity that will fit into x a total of n times.
  - Measuring model: We can think of dividing x into pieces of size y:
    $\frac{x}{y} = n$ is the number of times y will fit into x.

Addition & subtraction of fractions

If there is already a common denominator, add (or subtract) the numerators:

$$\frac{x}{z} + \frac{y}{z} = \frac{x+y}{z} \qquad \frac{x}{z} - \frac{y}{z} = \frac{x-y}{z}$$

If there is not a common denominator, first find one, then add (or subtract):

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

Operations and Precedence (order of operations)

1. Grouping (parentheses, radical bar, quotient bar)
2. Factorials,
3. Exponents and roots (top to bottom for exponents)
4. Multiplication and division (left to right)
5. Addition and subtraction (left to right)

Distributive Law

The one law that governs how the operations of addition and multiplication interact:

$$a(b+c) = ab + bc$$

Important implications of the distributive law that are often mistaken:

$$(x+y)(z+w) = xz + xw + yz + yw$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

Exponents

Suppose x is any real numbers and n is a natural number.

- Exponentiation: x^n is x multiplied together n times.
  That is, exponentiation can be viewed as repeated multiplication:

$$x^n = \underbrace{x \bullet x \bullet \cdots \bullet x}_{n \text{ times}}$$

Rules of exponents

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n} \qquad (x \neq 0)$$

$$\left(x^m\right)^n = x^{mn}$$

$$(xy)^m = x^m y^m$$

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m} \qquad (y \neq 0)$$

$$x^{-n} = \frac{1}{x^n} \qquad (x \neq 0)$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n \qquad (x, y \neq 0)$$

$$x^0 = 1 \qquad (x \neq 0)$$

0^0 is undefined

Roots

- $\sqrt{x}$ or $x^{\frac{1}{2}}$ is the square root of x
  the positive number whose square is x
- $\sqrt[n]{x}$ or $x^{\frac{1}{n}}$ is the n^{th} root of x
  the positive number whose n^{th} power is x