Algebra

Quantities are composed of
1. some object or system,
2. a quality of the object or system,
3. an appropriate unit or dimension, and
4. a process by which to assign numerical measure to the quality.

Variables are letters that may represent
- an arbitrary (generic) quantity
- a fixed (or unknown) constant
- a varying quantity
- a parameter (constant for a given situation but changes to a different constant when a different situation is considered)
- another mathematical object (function, point in space, vector field, etc.)

Dependence
- We often consider the variation in some quantities as depending on other quantities.
- A quantity *A* is dependent on another quantity *B* if knowing *B* is sufficient to determine *A*.
- A dependency may or may not be causal.
- A quantity can depend on multiple other quantities, each of which can depend on still others.

Proportionality
- Two varying quantities are proportional when
  1. The measure of one is always same multiple of the measure of the other.
     For example, $A = 2.7B$.
     In general, $A = kB$ (*k* is called the constant of proportionality).
  2. The measures of the two are always in the same ratio.
     For example, $A/B = 2.7$.
     In general, $A/B = k$
  3. Scaling one quantity by a factor results in a scaling of the other quantity by the same factor.
     For example, doubling *A* results in a doubling of *B*.
     In general, $A \to cA \quad \Rightarrow \quad B \to cB$
- If a quantity *A* is proportional to another quantity *B*, we write $A \propto B$.
- If $A \propto B$, then $\Delta A \propto \Delta B$ with the same constant of proportionality.
- If $A \propto B$, then $A = 0$ when $B = 0$. In other words, the graph goes through the origin.

Graphs
We often represent relationships between two variable quantities on a graph
- The independent quantity is represented as a horizontal distance.
- The dependent quantity is represented as a vertical distance.

Rate of Change

- If A is a varying quantity, we represent changes in A as ΔA.
  If the value of A changes from A_1 to A_2, the change is $\Delta A = A_2 - A_1$.
  If the value of A increases, then ΔA is positive.
  If the value of A decreases, then ΔA is negative.
  Note that by itself ΔA is ambiguous since it doesn't indicate exactly what change we mean.
- If A depends on B, then a specific change in B will correspond to a specific change in A.
- The average rate of change (of A with respect to B) on that interval is the ratio $\Delta A / \Delta B$.
- A rate of change $\Delta A / \Delta B$ is *constant* if for any ΔB, the average rate is the same.
- If the rate of change is not constant on an interval, the average rate is the constant rate for an imaginary situation which would produce the same ΔA for the given ΔB.
- Rate of change is often represented graphically as slope $\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$.

Algebraic Manipulations

Given an equation you can produce another equation that must also be true by

- Performing the same arithmetic operation to both sides
  (e.g., $2x - \pi = 3\pi \quad \Rightarrow \quad 2x - \pi + \pi = 3\pi + \pi$)
- Simplifying one side or both sides (so that their values do not change)
  (e.g., $2x - \pi + \pi = 3\pi + \pi \quad \Rightarrow \quad 2x = 4\pi$)
- Combine two equations
  - Substitute an entire expression for a variable
    (e.g., $\begin{matrix} A = \pi r^2 \\ r = 5t \end{matrix} \quad \Rightarrow \quad A = \pi\left(5t\right)^2$
  - Perform an operation using each side of the two equations
    (e.g., $\begin{matrix} a = b \\ x = y \end{matrix} \quad \Rightarrow \quad ax = by$)

Solving for a quantity x involves manipulating an equation until it is of the form

x = an expression without x's