Emerging Scholars
University of Texas

Fall 1997

WORKSHEET 32

1. Prove that the solutions to the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Hint: Complete the square.

2. Suppose h, k, and r are constants with $k/r > 0$.

a) Find the antidervative $\int \frac{1}{r(x+h)^2 + k}\,dx$ using the substitution $\sqrt{\frac{k}{r}}\tan\theta = x + h$.

b) Check your result by differentiating.

c) Suppose a, b, and c are constants with $b^2 - 4ac < 0$. Find $\int \frac{1}{ax^2 + bx + c}\,dx$.

3. Let s, p, and q be constants with $s \neq 0$ and $p \neq q$.

a) Show that $\frac{1}{s(x+p)(x+q)} = \frac{1}{s(p-q)}\left[\frac{1}{x+q} - \frac{1}{x+p}\right]$.

b) Find the antiderivative $\int \frac{1}{s(x+p)(x+q)}\,dx$.

c) Check your result by differentiating.

d) Suppose a, b, and c are constants with $b^2 - 4ac > 0$. Find $\int \frac{1}{ax^2 + bx + c}\,dx$.

4. Suppose a, b, and c are constants with $b^2 - 4ac = 0$. Find $\int \frac{1}{ax^2 + bx + c}\,dx$.

5. Summarize your results from Problems 2-4.

6. Evaluate the following integrals:

a) $\int \frac{x^2 - 1}{x^2 + 1}\,dx$ b) $\int \frac{x+1}{x+2}\,dx$ c) $\int \frac{x^3 + 1}{x^2 + x + 1}\,dx$

7. Find all continuous functions $f(x)$ which satisfy the equation

$$(f(x))^2 = \int_0^x f(t)\frac{t}{1+t^2}\,dt.$$