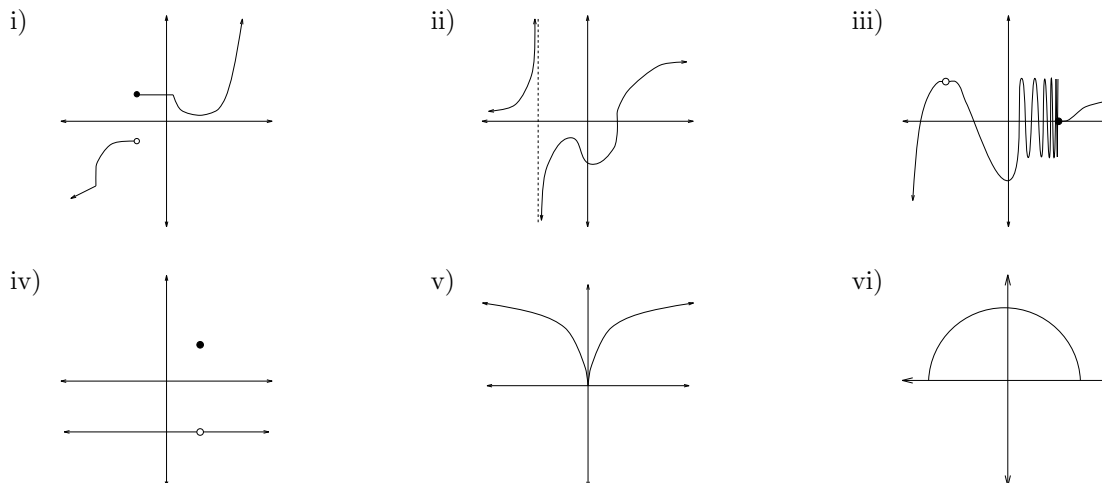


**WORKSHEET 11**

1. a) In each of the following graphs label all points of discontinuity and non-differentiability:



b) For each discontinuity, explain which parts of the definition of continuity are not satisfied.

c) For each point of non-differentiability, draw a blow-up of the graph at that point. Then draw a sequence (or two sequences if necessary) of secant lines which show that  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  does not exist.

If it is not possible to draw the appropriate secant lines, explain why.

2. Find the derivatives of the following functions:

a)  $f(x) = \sqrt[3]{x}$

b)  $f(x) = x^2$

c)  $f(x) = |4 - x^2|$

d)  $f(x) = \frac{x^2}{x^2 + 1}$

e)  $f(x) = \begin{cases} \frac{1-1/x}{1-1/x^2} & \text{if } x \neq 1 \\ \frac{1}{2} & \text{if } x = 1 \end{cases}$

f)  $f(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ -x & \text{if } x \leq 0 \end{cases}$

g)  $f(x) = \frac{x+2}{x^2-4}$

h)  $f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x^2 & \text{if } x \text{ is irrational} \end{cases}$

i)  $f(x) = \frac{1}{x+3} + \frac{1}{|x+3|}$

3. a) Sketch a graph of the six trig functions. Below each one, sketch a graph of its derivative using only graphical information.

b) Rewrite each of the six trig functions in terms of sine and cosine. Use these formulas and the quotient and product rules to compute the derivatives of all of the trig functions. Simplify and write the results next to the graphs of the appropriate derivatives.

c) Algebraically compute places where the derivatives are zero or are undefined. Label these points on both graphs.

4. Differentiate and simplify:

a)  $y = x\sqrt{1-x^2}$

b)  $y = x^2(1-x^2)^2$

c)  $y = (1 + \sqrt{x})(1 - \sqrt{x})$

d)  $y = \frac{x^2-1}{x^2+1}$

e)  $y = \frac{1-\sqrt{x}}{1+\sqrt{x}}$

f)  $y = \frac{x}{x^2+1}$

g)  $y = \frac{1}{(1-2x)^2}$

h)  $y = x^{2/3}(x+1)^{1/3}$

i)  $y = x(x+1)^{1/3}$

5. Rewrite each of the quotients in Problem 4 as a product. Then compute the derivatives using the product rule.