

WORKSHEET 7

1. Compute the following limits:

a) $\lim_{x \rightarrow 0} \frac{\tan^2 3x}{5x}$ b) $\lim_{x \rightarrow 1} \frac{\sin(x^2 - 1)}{x - 1}$ c) $\lim_{x \rightarrow -1} \frac{\sin(x^2 - 1)}{x - 1}$ d) $\lim_{x \rightarrow 0} \frac{\sin x^2}{\sin^2 2x}$

2. Find examples of functions f and g satisfying the following:

a) $\lim_{x \rightarrow c} g(x) = 0$ but $\frac{f(x)}{g(x)}$ does not have a vertical asymptote at $x = c$.

b) $\lim_{x \rightarrow c} f(x) = 0$ and $\frac{f(x)}{g(x)}$ does have a vertical asymptote at $x = c$.

c) $\lim_{x \rightarrow c} g(x) \neq 0$ and $\frac{f(x)}{g(x)}$ does have a vertical asymptote at $x = c$.

d) $\lim_{x \rightarrow c} f(x)$ exists and is not zero and $\lim_{x \rightarrow c} g(x) = 0$. Does the graph of $\frac{f(x)}{g(x)}$ have a vertical asymptote at $x = c$?

3. a) Find the slope of the secant line intersecting the graph of $f(x) = \sin x$ at x and $x + h$.

b) Simplify your expression using the identity

$$\sin(A + B) = \sin A \cos B + \sin B \cos A.$$

c) Compute the limit of these slopes as $h \rightarrow 0$.

4. Consider a fixed point (a, b) on the unit circle. Now let (x, y) represent a varying point on the unit circle.

a) Give algebraic conditions for (a, b) and (x, y) to lie on the unit circle.

b) Find the slope of the secant line between these two points.

c) What happens to your expression for the slope of the secant line as $(x, y) \rightarrow (a, b)$?

d) Find the slope of the tangent line to the circle at (a, b) .

Hint: Multiply by a clever form of one which makes the numerator a difference of squares. Then use your equations from part a) to rid the numerator of all b 's and y 's. Then...

5. Let $f(x) = \begin{cases} 1 & x \geq 2; \\ x & x < 2. \end{cases}$

a) Find a formula giving the slope of the secant line intersecting $f(x)$ when $x = 2$ and $x = 2 + h$.

Note: You will have different answers for $h < 0$ and $h > 0$. Why?

b) Find the limit of these slopes as $h \rightarrow 0^-$ and $h \rightarrow 0^+$.

6. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket coasts upward for awhile and then begins to fall. A small explosive charge pops out a parachute shortly after the rocket starts down. The parachute slows the rocket to keep it from breaking when it lands.

The figure to the right shows velocity data from the flight of a model rocket. Use the data to answer the questions below.

a) How fast was the rocket climbing when the engine stopped?

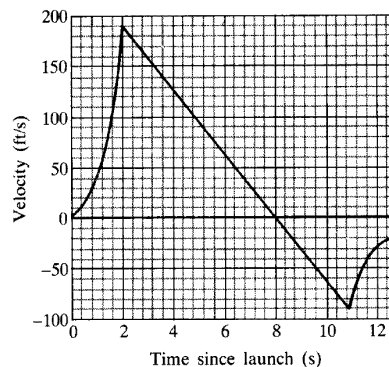
b) For how many seconds did the engine burn?

c) When did the rocket reach its highest point? What was its velocity there?

d) When did the parachute pop out? How fast was the rocket falling then?

e) How long did the rocket fall before the parachute opened?

f) Very carefully sketch a graph of the height of the rocket (in feet) versus time (in seconds).



Velocity of a model rocket