WORKSHEET 39
a.k.a. Practice Exam 3

1. Compute $\displaystyle\int_2^\infty \frac{x^3 - x^2 - 2}{x^4 - x^3 + x^2 - x}\,dx$

2. Graph $r = \cos 4\theta$ in polar coordinates. Find the area of the region bounded by this graph.

3. An object is moving along the path $x = 50t$, $y = 16 - 16t^2$.
   a) What is its speed at $t = 1$?
   b) At what time is the object moving fastest?
   c) How far does the object travel in the upper half plane (i.e., where $y > 0$)?

4. For what values of p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge? Justify your answer.

5. Show that the Taylor series for $\sin x$ converges for all real numbers and is equal to $\sin x$.

6. Consider the plane parallel to the vectors $(1, 1, 1)$ and $(2, 0, 1)$ containing the point $(0, 1, 0)$. Find the point on this plane closest to the origin.

7. Prove that if the speed of a particle is constant, $\mathbf{a}$ is perpendicular to $\mathbf{v}$.

8. Let $f(x, y)$ be a differentiable function of two variables and $\mathbf{u}$ a unit vector in the (x, y)-plane. Give the definitions of $\partial f/\partial x$, $\partial f/\partial y$, and $D_{\mathbf{u}} f$, and then describe how you would actually compute each of these.

9. Let $w = f(x - y, y - z, z - x)$. Show that $\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z} = 0$.

10. Let $T(x, y, z)$ be the temperature at the point (x, y, z). Assume that ∇T at $(1, 1, 1)$ is $\langle 2, 3, 4\rangle$.
    a) Find $D_{\mathbf{u}} T$ at $(1, 1, 1)$ if $\mathbf{u}$ is in the direction of the vector $\langle 1, -1, 2\rangle$.
    b) Estimate the change in temperature as you move from the point $(1, 1, 1)$ a distance 0.2 in the direction of the vector $\langle 1, -1, 2\rangle$.
    c) Find three unit vectors $\mathbf{u}$ such that $D_{\mathbf{u}} T = 0$ at $(1, 1, 1)$.

11. Evaluate $\displaystyle\int_0^1 \int_x^1 \sin y^2\,dy\,dx$ and $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \frac{dy}{\sqrt{1 + y^3}}\,dx$.

12. Find the center of mass of the region bounded by $y = x$ and $y = x^2$ in the first quadrant with density at (x, y) given by e^{-x}.

13. Find the moment of inertia about the origin of the region in Problem 12.

14. Find the average of the function $f(P)$ over the region within the cardioid $r = 1 + \cos\theta$ where $f(P)$ is the distance from the point P to the x axis.

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