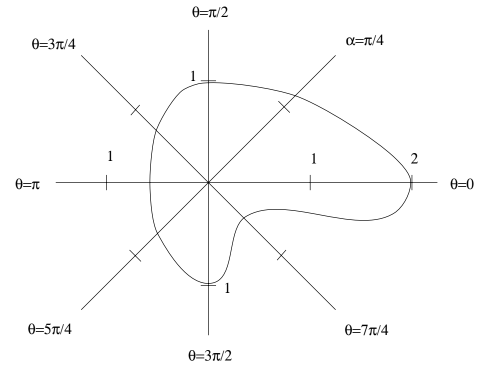
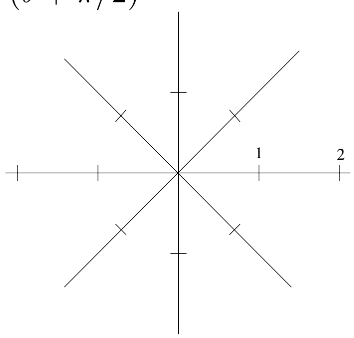
WORKSHEET 25

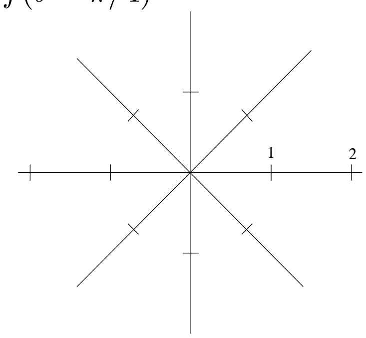
1. Below is a graph, in polar coordinates, of $r = f(\theta)$. On each of the six axes provided, sketch a graph of the modified function indicated.



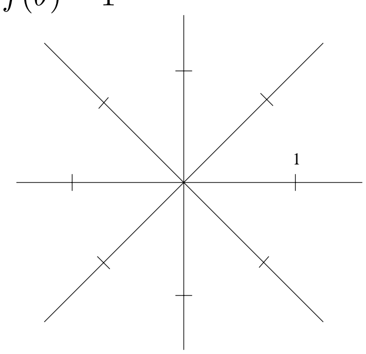
a) $r = f(\theta + \pi/2)$



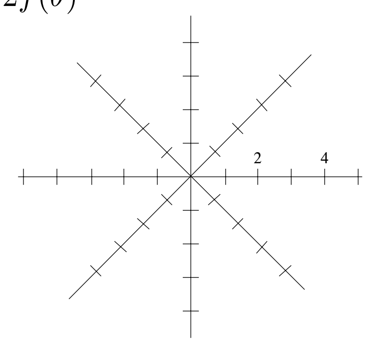
b) $r = f(\theta - \pi/4)$



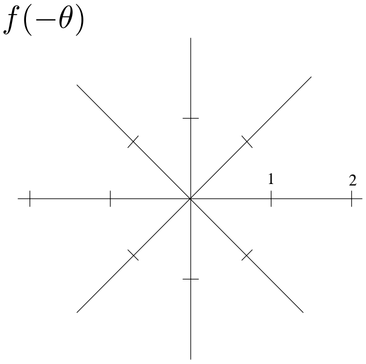
c) $r = f(\theta) - 1$



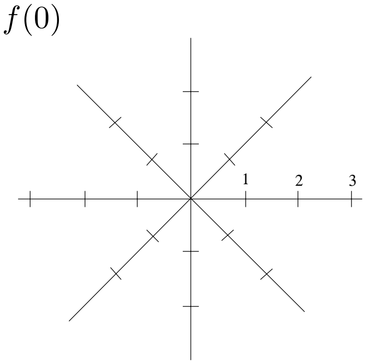
d) $r = 2f(\theta)$



e) $r = f(-\theta)$



f) $r = f(0)$



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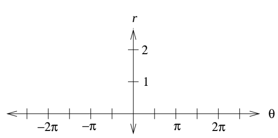
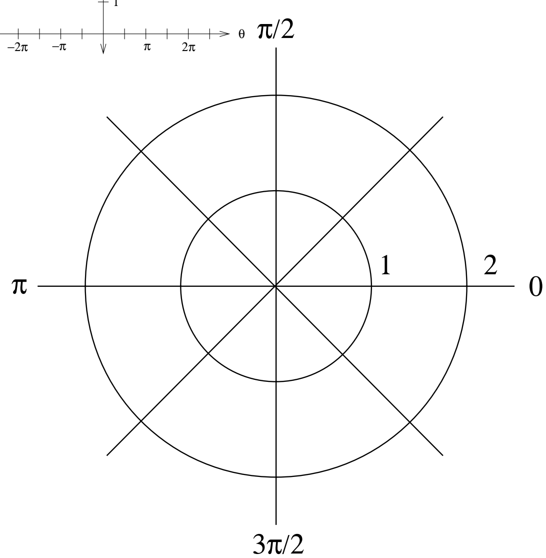
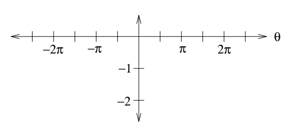
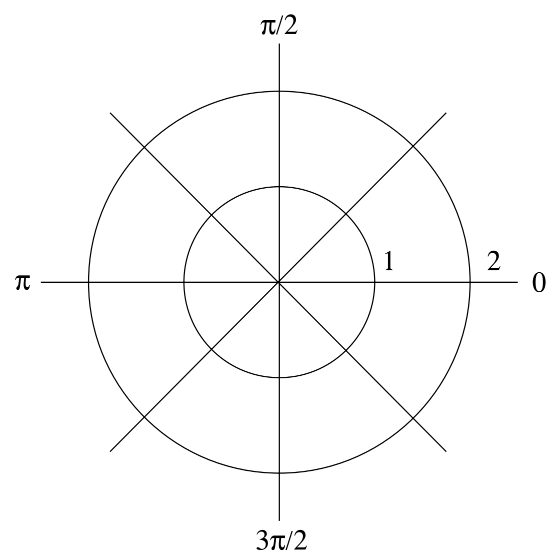
2. Graph each function below in two different ways: first plot r on the vertical axis as a function of θ on the horizontal axis, then plot r at a radial distance from the origin as a function of the angle θ from the positive x-axis.

   a) $r = 1 + \cos\theta$ b) $r = \cos\theta - 1$.









3. If two points have polar coordinates (r_1, θ_1) and (r_2, θ_2), show that the distance d between them is given by

$$d^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2).$$

What does this mean geometrically?

4. Write down the Maclaurin series for $\sin x$, $\cos x$, and e^x. Prove that $e^{i\theta} = \cos\theta + i\sin\theta$. Using this formula, express the given numbers in the form $re^{i\theta}$ for a positive real number r and argument θ, $-\pi < \theta \leq \pi$:

   a) $\dfrac{e^2}{\sqrt{2}} - \dfrac{e^2}{\sqrt{2}}i$

   b) $3\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

   c) $5\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right) \cdot 3\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

   d) $7\left(\cos\dfrac{7\pi}{3} + i\sin\dfrac{7\pi}{3}\right)$

5. Find the intersection of the curve $r = 1 - \cos\theta$ and the circle $r = \cos\theta$.