

WORKSHEET 16
a.k.a. Practice Exam 1

1. Does the series $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$ converge absolutely, converge conditionally, or diverge?

2. Find $\lim_{x \rightarrow 0} x \ln x$.

3. Consider the Fibonacci sequence which is defined by $F_0 = 1$, $F_1 = 1$, and

$$F_{n+2} = F_{n+1} + F_n, \quad n \geq 2.$$

Find the limit

$$\lim_{n \rightarrow \infty} \frac{F_{n+2}}{F_n}.$$

4. Compute $\int_{-\infty}^0 x^2 e^x dx$.

5. Compute $\int \frac{x-1}{x^2+1} dx$.

6. Find the value of $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n$.

7. a) Find the Taylor series for $f(x) = \ln x$ centered at $x = 1$.

b) What is the interval of convergence for this series?

c) What is $P_5(x; 1)$?

8. Prove $\lim_{n \rightarrow \infty} \frac{1}{n+2} = 0$.

9. Find values for a and b such that the power series $\sum_{n=1}^{\infty} \frac{a^n(x+b)^n}{n^2}$

converges on $(-3, 1)$ and diverges for $x < -3$ and $x > 1$. Then determine what happens at the endpoints $x = -3$ and $x = 1$.

10. Find the volume of the solid whose base is a circle of radius r and with perpendicular cross-sections equilateral triangles.