WORKSHEET 8

1. Consider the *harmonic series*

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \cdots$$

which we have seen is divergent, that is, the partial sums tend to infinity. But you might ask at what rate do these partial sums get large?

a) Recall that John's computer program added roughly one billion terms in 30 minutes (to a grand total of 21.3). Suppose that you started this program computing the partial sums with $s_1 = 1$ eleven billion years ago (one estimate of the age of the universe). Assuming the computer never loses accuracy, how large would s_n be today? Guess.

b) Last week we discovered a way to bound the partial sums of a series (either above or below) by a corresponding integral. Taking a limit allowed us to test for convergence or divergence of certain series by looking at what happened to the resulting improper integral. We called this the **integral test**. Use these concepts to compute upper and lower bounds for the partial sum s_n of the harmonic series.

c) Check your guess to part a).

d) Can you determine how much accuracy John's computer lost in computing the one billionth partial sum?

2. You know that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (Do you remember why?), but to what number does it converge? This is a difficult, and thus for the purposes of this worksheet hypothetical, question. Compute the partial sums out to several terms. How close do you think these are to the actual limit? Be clever and devise a method to find out how many terms you would need to add to ensure that your estimate will be off by not more than .001.

3. a) Show that $\lim\limits_{c\to\infty} \int_{-c}^{c} x\,dx$ exists. **Find it.**

b) Explain why the limit in a) exists however $\int_{-\infty}^{\infty} x\,dx$ diverges.

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4. Let $f\colon [0,\infty) \to \mathbb{R}$ be a function with $f(0) = 2$ and $\lim_{x\to\infty} f(x) = 3$. Find the following:

   a) $\displaystyle\lim_{b\to\infty} \frac{1}{b}\int_0^b f(x)\,dx$ b) $\displaystyle\int_0^\infty f(x)\,dx$ c) $\displaystyle\lim_{b\to 0} \frac{1}{b}\int_0^b f(x)\,dx$

5. A working light bulb is in a closed room with no windows. Outside the room, is a panel of three switches, one of which controls the light inside (up is on, down is off.) You may do anything you like to the three switches and then enter the room to inspect the light. After this, without any further experimentation, you must indicate which switch controls the light. What do you do?

6. a) What is a reasonable value to assign to the unending expression

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}}$$

   b) What is a reasonable value to assign to the unending expression

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}}$$