WORKSHEET 3

1. Compute the following indefinite integrals:

   a) $\int x \cos x \, dx$ b) $\int x^2 e^x \, dx$ c) $\int e^x \cos 3x \, dx$

   d) $\int \sqrt{x} \ln x \, dx$ e) $\int x^3 e^{x^2} \, dx$ f) $\int \sin(\ln x) \, dx$

2. Let f and g be differentiable functions. Find

$$\int f(x)g'(x) \, dx + \int g(x)f'(x) \, dx.$$

3. Compute

   a) $\displaystyle\int_{-1}^{0} \frac{2x}{(x^2+1)(x-1)^2} \, dx$

   b) $\displaystyle\int \frac{x^2}{x^2-1} \, dx$

4. Sociologists sometimes use the phrase *social diffusion* to describe the way information spreads through a population. The information might be a rumor, a cultural fad, or news about a technical innovation. In a sufficiently large population, the number of people x who have the information is treated as a differentiable function of time t. The rate of diffusion, dx/dt, is assumed to be proportional to the number of people who have the information times the number of people who do not. This leads to the differential equation

$$\frac{dx}{dt} = kx(N-x),$$

   where N is the number of people in the population.
   Suppose t is measured in days, $k = 1/250$, and two people start a rumor at time $t = 0$ in a population of $N = 1000$ people.

   a) Find x as a function of t.

   b) When will half the population have heard the rumor?

   c) When will the rumor be spreading the fastest?

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5. Many chemical reactions are the result of interaction of two molecules that undergo a change to produce a new product. The rate of the reaction typically depends on the concentrations of the two kinds of molecules. If a is the amount of substance A, and b is the amount of substance B at time $t = 0$, and if x is the amount of product at time t, then the rate of formation of x may be given by the differential equation

$$\frac{dx}{dt} = k(a - x)(b - x),$$

or

$$\frac{1}{(a - x)(b - x)} \frac{dx}{dt} = k$$

where k is a constant specific to the reaction. Integrate both sides of this equation with respect to t to obtain a relation between x and t

a) if $a = b$, and

b) if $a \neq b$.

Assume in each case that $x = 0$ when $t = 0$. What does this assumption mean physically?

6. A catalyst for a chemical reaction is a substance that controls the rate of the reaction without undergoing any permanent change in itself. An autocatalytic reaction is one whose product is a catalyst for its own formation. Such a reaction may proceed slowly at first if the amount of catalyst present is small, and slowly again at the end when most of the original substance is used up. But in between, when both the substance and its product are abundant, the reaction may proceed at a faster rate. In some cases it is reasonable to assume that ν is proportional to $ax - x^2$ where

$x =$ the amount of product,
$a =$ the amount of substance at the beginning.

The equation that describes the autocatalytic reaction can be written as

$$\frac{dx}{dt} = kx(a - x).$$

Solve this equation to find x as a function of t. Assume that $x = x_0$ when $t = 0$.