Math 300: Homework Problems Name:

1. An investor intends to buy shares in 3 different companies chosen from a list of 21 companies recommended by his broker. How many different options are there under the following circumstances?

   a. Equal amounts will be invested in the chosen companies.

   b. Amounts of $10000, $25000 and $30000 will be invested in the chosen companies.

2. Suppose that 4 freshmen, 7 sophomores, 3 juniors, and 11 seniors have been nominated to serve on a student advisory committee. How many different committees can be formed under the following circumstances?

   a. The committee is to consist of any four persons.

   b. The committee is to consist of one member from each class.

   c. The committee is to consist of two persons: one freshman or sophomore and one junior or senior.

   d. The committee is to consist of three persons, each from a distinct class.

3. An office worker is purchasing snacks for a morning meeting. He may buy donuts in one dozen increments or cookies in one pound packages. There are 8 varieties of donuts available and he may make up a one dozen box of donuts in any way he wishes. There are 30 varieties of cookies available, and a package consists entirely of one variety.

   a. How many different outcomes are possible if he buys one dozen donuts or 2 packages of cookies, but not both?

   b. How many outcomes are possible if he buys both a dozen donuts and 2 packages of cookies?

   c. How many outcomes are possible if he buys both a dozen donuts and 2 packages of cookies and he must include at least 3 glazed donuts in the box?

4. A convention hall contains 112 tables and 837 chairs. If every chair is placed at some table, what is the smallest possible number of chairs at the table having the most chairs?

5. Four players are dealt 5 cards each from a standard deck of 52 cards. The order in which a particular player's cards are dealt does not matter, (but which player receives them does!). How many different outcomes are possible?

6. Prove that $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{k}$ for every positive integer n.