GREENBERG'S CONJECTURE FOR ABELIAN NUMBER FIELDS

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Abstract. Two different approaches to Greenberg's conjecture will be discussed. The criterion of Ichimura and Sumida gives an equivalent condition for Greenberg's conjecture in terms of the arithmetic at the n-th level in the cyclotomic $\mathbb{Z}_p$-extension and cyclotomic units. After a brief introduction to Iwasawa theory and Greenberg's conjecture, I will discuss the criterion and possible ways to prove that it holds. The second approach of Sumida involves approximating the Galois group of the maximal p-ramified p-extension of k_∞ by an elementary Λ-module E. If the approximation is close enough, Galois groups of certain p-extensions of k_n will correspond to quotients of E, and this correspondence can be used to ascertain that Greenberg's conjecture holds for a character and prime p.