Spatial Simulation Lab

Benjamin Morin
Adapted from labs from David E. Hiebeler

1. Spatial S-I-R Model

We will simulate a continuous-time spatially explicit S-I-R epidemiological model. We will be modeling localized interactions.

The basic model is as follows:

- The simulation will take place in an $L \times L$ lattice, with a total of $N = L^2$ sites.
- Each site may be susceptible, infectious, or resistant.
- Each infectious site will contact other sites at rate β. (It will choose the other site by selecting one of the target's four neighbors with wrap around). If the contacted site is susceptible, it will become infectious (change its state from S to I).
- Each infectious site will permanently recover at rate α; when doing so, it changes its state to R.

If we think about it, infectious sites are the only ones which are the sources of events (even though the events may affect susceptible sites or produce resistant sites), we really only need to do the "choose a site at random from among all sites in a certain state" for infectious sites. So we will keep vectors of the x and y coordinates of the infectious sites, not for the susceptible or resistant sites. We will keep a matrix (called **stateArray**) containing the values 0, 1, and 2 representing susceptible, infectious and resistant sites, respectively. The vectors containing the x and y coordinates of all the infectious sites will be called **Ixv** and **Iyv** respectively.

Because we are being spatial we may lose infection attempts (i.e. an infected site attempting to infect an already infected or recovered site) there will be a lot of events that we can ignore so we will not use EVERY little value in our time series when we display. We will maintain vectors **currentS**, **currentI**, and **currentR**. Then for every $\Delta t = .05$ that passes we will store the values that describe the state of our system (don't worry this feature of the code is already coded).

The model will have one more generalization. There will be a variable **proportionLongDistance**, call it λ, that determines how often an infection attempt is mean field. Thus, for each infection attempt, $1 - \lambda$ will be the proportion of those infectors that look only to its nearest 4 neighbors. The rest will choose a site at random from the entire lattice. We will study the model on a 100×100 lattice with $\beta = 6.2$, $\alpha = 1.5$, **numI0** $= 50$.

♡ 1. Turn in the parts of the code that you had to supply.

♣ 2. With **proportionLongDistance**$= 1$, all the infections are long distance, run the simulation. Turn in a plot of S, I, and R versus t. What are the final values of S, R and t? (You can just look at the last values of S, R, and et for this.) Also, what is the maximum number of infected individuals? (**max(I)** should provide this.)

♣ 3. Go up to the top of the code and change **proportionLongDistance** to be 0. This will make all the interactions local! Again pass in the plot of S, I, and R versus t and give the final values of S, R, and t as well as the maximum value of I.

♣ 4. Set **proportionLongDistance** to .5 and pass in all the same information as 2. and 3. Is the behavior here closer to that of pure long-distance interactions or those of the pure local interactions.

♢ 5. Set **proportionLongDistance** to .1 (90% of the interactions are local and 10% are long-distance). Again turn in the same graph and calculated values. How does the amount of long-distance interactions seem to affect the dynamics?

♡ 6. Numerically integrate the mean field $S - I - R$ equations

$$\frac{dS}{dt} = -\beta S \frac{I}{N} \tag{1}$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \alpha I \tag{2}$$

$$R = N - S - I \tag{3}$$

Where $N = 10,000$, $I(0) = 50$, $\alpha = 1.5$, and $\beta = 6.2$. Let time go "long enough" so that the I population is pretty much empty (i.e less than 1), but don't let time go on "too long". Turn in a plot of S, I, and R versus t. What is the final value for S, R and t? What is the maximum value of I? Compare this to your previous results.

♠ 7. BONUS! Numerically integrate the $S - I - R$ pair approximation equations. This is like the $S - I - R - S$ pair equations from the homework but without the $R \to S$ transition. Figure out initial conditions that match those of the lattice model. Turn in a plot of S, I, and R versus t. What is the final value for S, R and t? What is the maximum value of I? Compare this to your previous results.