Samira Ejaz
Brewer
MAT266
11 May 2010

Colin Maclaurin: The Maclaurin Series

Colin Maclaurin is one of many famous mathematicians known for his great contributions to the field of mathematics that are still used today. Maclaurin was born in February 1698, in Scotland and as he grew older, it was clear that he was a child prodigy with an extreme affinity for mathematics. At the age of eleven, he was admitted into the University of Glasgow.

It was at this university that he first encountered advanced mathematics when he found Euclid's *Elements*. He chose to study these books on his own, although they were the standard text in mathematics. He mastered many of the books. At the age of 14, he obtained his degree after defending a thesis which focused on the power of gravity. This thesis played a large part in developing Newton's theories.

Still a youth, he became a mathematics professor at Marischal College at age nineteen. He traveled to London to meet Newton and was elected a Fellow of the Royal Society when he was there. He met many French mathematicians as well and was awarded a Grand Prize by the Academy of Sciences in Paris. During his lifetime, he collected many awards and prizes for his notable work towards the field of mathematics such as geometry and others. He also made publications of some of his work.

Maclaurin obtained his desired position at University of Edinburgh in 1725, the place where he spent the rest of his career. He married in 1733. Soon afterwards, he tried to expand the Medical Society of Edinburgh. In doing so, he created a society where he would read contemporary works of the time on scientific developments. This society later became the Royal Society of Edinburgh after his death.

Although he developed many theorems and proofs, he is very famous for the special case of the Taylor series, which is named after him in his honor. Another one of his major contributions to the field was the integral test for the convergence of an infinite series as well as giving many applications of calculus. However, the Maclaurin series is the work that he is best remembered by today. It is used to approximate function values, much more effectively than standard linear approximations.

The Mclaurin series is the Taylor series at a=0. The Taylor series is as follows:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1} \quad , \qquad |x-a| < R$$

If f has n+1 derivates in a n interval I that contains the number a, then for x in I there is a number z strictly between x and a such that the remained term in the Taylor series can be expressed as above.

Plugging in a=0 gives:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n+1}(z)}{(n+1)!}(x)^{n+1} \quad , \qquad |x| < R$$

which is essentially the Maclaurin series.

Thus, the Maclaurin series can be evaluated the same way as Taylor series. However, there are some special formulas that are only applicable to Maclaurin series. They are as follows:

$$e^x = \sum\nolimits_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos \chi = \sum\nolimits_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin \chi = \sum\nolimits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\tan^{-1} \chi = \sum\nolimits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$