The Calculus behind a Basketball Shot

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Integral calculus can be incorporated into daily life in many ways. All around us, mathematical concepts are being applied to everything we do. Basketball is one sport that can easily be related to the different concepts of integral calculus. One of the most important parts of the game of basketball, shooting the ball, can be described through arc length equations.

The path taken by a basketball when shot can be split into two components, the horizontal (x) direction and the vertical (y) direction. These two components can be represented by the parametric equations:

$$x(t) = x_o + v_o \cos(\theta) t$$

$$y(t) = y_o + v_o \sin(\theta) t + \frac{1}{2} g t^2$$

The variables are considered to be;

x_o is the initial horizontal position of the basketball.

y_o is the initial vertical position of the basketball.

v_o is the initial velocity of the basketball.

θ is the angle the ball is projected with respect to the x-axis.

g is the acceleration due to gravity, -9.81 m/s^2.

t is the time traveled.

In order to fully understand the path of a basketball, one must consider the situation of a person shooting a basketball. Imagine a basketball court as a coordinate plane system where the shooters feet rest at the origin, (0,0), and the basketball hoop is located a distance, d, away from the shooter. A regulation hoop stands 10 feet high or 3.05 meters high, therefore the final destination of the basketball is the point (d,3.05). Using my height of 6 feet 4 inches and an average jump when shooting of 8 inches the release point of the ball would be 7 feet or 2.13 meters in the air, point (0,2.13). Inputting the data into the two parametric equations the equations change to:

$$x(t) = v_o \cos(\theta) t$$

$$y(t) = 2.13 + v_o \sin(\theta) t - 4.905 t^2$$

With the new revised equations, the distance the basketball travels can be found using the arc length equation, $L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt, \alpha \le t \le \beta$. The derivatives of x(t) and y(t) with respect to time *t* are:

$$\frac{dx}{dt} = v_o \cos(\theta)$$

$$\frac{dy}{dt} = v_o \sin(\theta) - 9.81t$$

Therefore

$$L = \int_{\alpha}^{\beta} \sqrt{(v_o \cos(\theta))^2 + (v_o \sin(\theta) - 9.81t)^2} =$$

$$\int_{\alpha}^{\beta} \sqrt{v_o{}^2 \cos^2(\theta) + v_o{}^2 \sin^2(\theta) - 19.62 * t * v_o \sin(\theta) + 96.24t^2}\, dt =$$

$$\int_{\alpha}^{\beta} \sqrt{v_o{}^2 - 19.62 * t * v_o \sin(\theta) + 96.24t^2}\, dt$$

Using the average velocity of a basketball shot, 2.24 m/s, the shot angle that would produce maximum efficiency, 45 degrees, and the time it would take the ball to travel from the free throw line, about 2 s, the arc length can be calculated.

$$L = \int_{0}^{2} \sqrt{2.24^2 - 19.62 * t * (2.24 \sin(45) + 96.24t^2}\, dt = 17.34$$