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Footnote 18

Calculus for the Gold

----Looking at the Calculus behind Swimming----

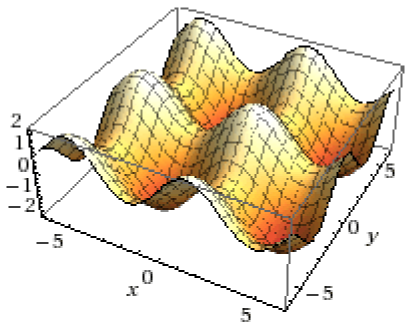
In the 2008 Beijing Summer Olympic Games Michael Phelps caught the attention of the whole world when he shattered world records and obtained enough gold medals to be celebrated as one of the greatest recorded Olympians of all time, with a record 14 Gold Medals. With all the hype around the victories of Michael Phelps it begs the question: Why him? Is there anything about Phelps that gives him an advantage over others? It could in fact be that his wingspan is three inches longer than his height, measuring in at around 6'7". It could also be that he has a shoe size of 14, which is considerably larger than the average size for his height. Or is it that he has a larger than average hand size? In this paper I will attempt to evaluate the advantages of Michael Phelps through the utilization of multivariable Calculus applied to competitive swimming. More specifically I will consider the average hand size of an individual and compare how much surface area that hand can cover in the water when compared to an above-average hand size like the hand size of Michael Phelps.

Before starting the project certain assumptions must be made about the average hand sizes in the United States. Following is information derived from a U.S. Government made chart that charts out average hand sizes for Men in the U.S. This information is essential for the project and contains the Hand length of a hand fully extended and the length of the hand slightly arced, this arched version is the one that will be used as it is the more appropriate swimming style. The arched version was found by simple taking off .5 inches or 2.55 cm from the total hand length.

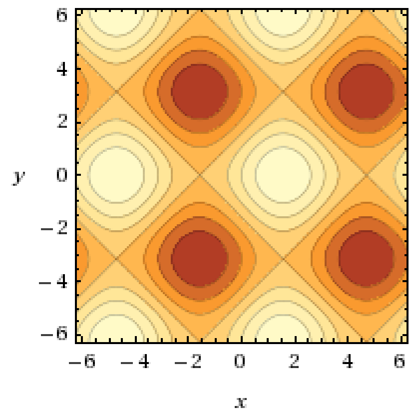
	50^{th} Percentile	95^{th} Percentile
Hand Length	19.3 cm	21.1 cm
Hand Length (Projection with arched hand)	16.45 cm	18.55 cm
Hand width	9.0 cm	9.8 cm

http://www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA467401&Location=U2&doc=GetTRDoc.pdf

Now with this information one can create an ellipse that will help us get the appropriate variables needed for calculation purposes. In order to start calculations one must determine an appropriate graph for a 3 dimensional water surface that will act as the surface being covered by the palm. The equation used for the surface was: $z = \sin(x) + \cos(y)$



3D Plot of surface



Contour Plot of surface

(Graphs above computed using http://www.wolframalpha.com)

From here we can do the following computations to find the surface area of the water covered by an average sized hand and an above average sized hand. Following are some general equations:

$$General\ equation\ of\ projection: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x = \mathrm{acos}(t)$$

$$y = \mathrm{bsin}(t)$$

$$0 \leq t \leq 2\pi$$

$$Surface\ Area: \qquad A(S) = \iint_D \sqrt{1 + {Z_x}^2 + {Z_y}^2}\, dA$$

$$Greens\ Theorem: \qquad A(S) = \iint_D (Q_x - P_y)\, dA = \oint_C P dx + Q dy$$

Average Hand Size Calculations:

$$\frac{x^2}{8.23^2} + \frac{y^2}{4.5^2} = 1$$

$$x = 8.23\cos(t)$$

$$y = 4.5\sin(t)$$

$$0 \leq t \leq 2\pi$$

$$z = \sin(x) + \cos(y)$$

$$Z_x = \cos(x) \quad , \quad Z_y = -\sin(y)$$

$$Surface\ Area: \qquad A(S) = \iint_D \sqrt{1 + (\cos x)^2 + (-\sin y)^2}\ dA$$

$$Surface\ Area: \qquad = A(S) = \iint_D \sqrt{2}\, dA \qquad\qquad Q_x = \sqrt{2} \qquad Q = \sqrt{2}x$$

$$P_y = 0 \qquad P = 0$$

$$A(S) = \iint_D (\sqrt{2} - 0)\ dA = \oint_C Pdx + Qdy = \int_0^{2\pi} \sqrt{2}\, x\ dy$$

Where x and y are in terms of t:

$$x = 8.23\cos(t)$$

$$y = 4.5\sin(t)\,, \qquad dy = 4.5\cos(t)$$

$$= \int_0^{2\pi} \sqrt{2}\,(8.23\cos t)(4.5\cos t)dt = 37.035\sqrt{2}\int_0^{2\pi} \cos^2 t\, dt$$

$$= 37.035\sqrt{2}\int_0^{2\pi} \frac{1}{2} + \frac{\cos 2t}{2}\ dt = 37.035\sqrt{2}\ \left[\frac{1}{2}(2\pi) + \frac{sin4\pi}{4} - (0+0)\right] = 52.38\pi\ cm^2$$

Average Hand Size Calculations:

$$\frac{x^2}{8.23^2} + \frac{y^2}{4.5^2} = 1$$

$$x = 9.28\cos(t)$$

$$y = 4.9\sin(t)$$

$$0 \le t \le 2\pi$$

$$z = \sin(x) + \cos(y)$$

$$Z_x = \cos(x) \quad , \quad Z_y = -\sin(y)$$

$$Surface\ Area: \qquad A(S) = \iint_D \sqrt{1 + (\cos x)^2 + (-\sin y)^2}\ dA$$

$$Surface\ Area: \qquad = A(S) = \iint_D \sqrt{2}\, dA \qquad Q_x = \sqrt{2} \qquad Q = \sqrt{2}x$$

$$P_y = 0 \qquad P = 0$$

$$A(S) = \iint_D (\sqrt{2} - 0)\ dA = \oint_C Pdx + Qdy = \int_0^{2\pi} \sqrt{2}\, x\, dy$$

Where x and y are in terms of t:

$$x = 9.28\cos(t)$$

$$y = 4.9\sin(t)\,, \qquad dy = 4.9\cos(t)$$

$$= \int_0^{2\pi} \sqrt{2}\,(9.28\cos t)(4.9\cos t)dt = 45.47\sqrt{2}\int_0^{2\pi} \cos^2 t\, dt$$

$$= 45.47\sqrt{2}\int_0^{2\pi} \frac{1}{2} + \frac{\cos 2t}{2}\ dt = 45.47\sqrt{2}\left[\frac{1}{2}(2\pi) + \frac{sin4\pi}{4} - (0+0)\right] = 64.31\pi\ cm^2$$

To conclude my report, when looking at the calculations one notices that for an average hand, the surface area that it covers in the water is approximately$52.38\pi\ cm^2$. This is compared to the surface area that an above average hand like that of Michael Phelps has in the water which is $64.31\pi\ cm^2$. In looking at this comparison Michael Phelps clearly has an advantage when it comes to the amount of water that he is able to push with his hand. From the information that I gathered I would be able to go more in depth and actually show the specifics about how much the average hand versus the above-average hand can push in the water given their respective surface areas. For now my project is complete as I was able to successfully conclude why Michael Phelps has an edge over other swimmers. I utilized certain concepts in Calculus III such as Greens theorem in order to substantiate the claim that it is easier to push through the water with above average. In completing this project I learned that even in the most practical applications Calculus can be found. My application of Calculus to swimming has given me a newfound awareness of the technical aspects behind competitive swimming.