Brewer MAT 275 – Spring 2009
Final Exam
Name:

1. Find the general solution (explicit if possible) of the differential equation: **[8 pts]**
   $y' = y \sin x$

2. Find the particular solution (explicit if possible) of the differential equation: **[8 pts]**
   $3xy' + y = 12x \qquad y(1) = 4$

3. Apply Euler's method to approximate the solution to $y(1)$ on the interval [0,1] with h = 0.5. Compare the error at each step to the given exact solution: **[8 pts]**
   $y' = 2xy,\quad y(0) = 1,\qquad$ The exact solution is $y(x) = exp(x^2)$

n	x_n	y_n	$f(x_n, y_n)$	$y_{n+1} = y_n + h \cdot f(x_n, y_n)$	*Error*

4. Calculate the Wronskian for the given equations. Are the equations linearly independent or linearly dependent? $f(x) = e^x \cos x + e^x \sin x \quad g(x) = 3\cos x - 2\sin x$ **[5 pts]**

5. Find the general solution of the differential equation by solving the characteristic equation: **[8 pts]**
$y^{(4)} - 8y^{(3)} + 16y'' = 0$

6. When a mass is attached to a spring and no damping constant, the resulting position function is $x(t) = 5\cos(6t) - 12\sin(6t)$. Write the position function in the form $x(t) = C\cos(\omega_0 t - \alpha_0)$. **[8 pts]**

7. Find the solution of $y'' + 3y' + 2y = e^x$ with $y(0) = 0$, $y'(0) = 3$ using the method of undetermined coefficients. **[10 pts]**

8. Find a particular solution to problem 7 using the method of variation of parameters. **[8 pts]**

9. Solve the system of differential equations: **[8 pts]**
   $x' = x + 2y,$
   $y' = 2x + y$

10. For problem 9, find: **[6 pts]**

a) The fundamental matrix, $\Phi(t)$

b) The exponential matrix, $e^{t\mathbf{A}} = \Phi(t).\ \Phi(0)^{-1}$

c) The solution by multiplying $X(t) = e^{t\mathbf{A}}[3 \;\; 5]^T$

11. Find the inverse Laplace transform of : **[8 pts]**

$$R(s) = \frac{(s^2 + 1)}{s(s^2 - 2s - 8)}$$

12. Find the Laplace transform of : **[5 pts]**

$f(t) = t \sin(3t)$

13. Solve the initial value problem by taking the Laplace Transform: **[10 pts]**

$y'' - 7y' - 8y = u(t-2), \qquad y(0) = 0, \quad y'(0) = 0$