FN 18 Project:

Transient Circuit Analysis: The Differential Equations Approach

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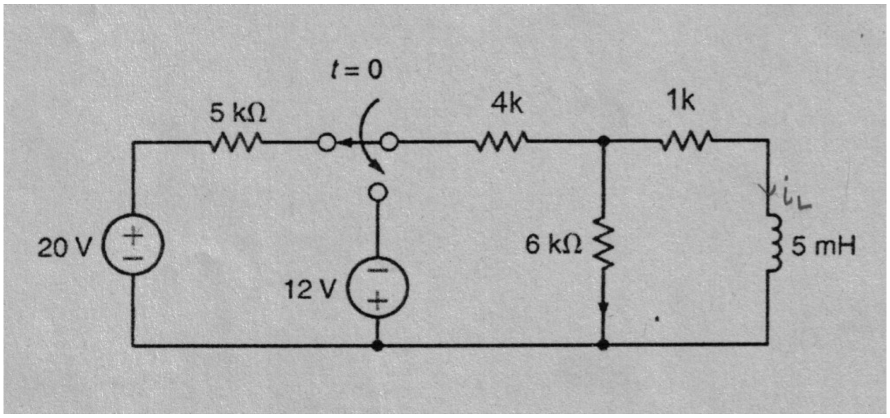
Brewer, MAT 275: Fall 2009



Calculate the inductance current (I_L).

This is a first-order transient circuit (presence of the switch) containing a 5mH inductor. The switch is open at t = 0 such that the 20V DC source is connected to the inductor. After the switch is flipped (closed), the 20V source and 5 kΩ resistor no longer have current going through them and can be, at that point, ignored as the inverted 12V DC source is connected to the inductor.

<u>Motivation</u>: The differential equation approach has more application than the step-by-step method of solving transient circuits, as it can be applied to second and higher order LRC Circuits whereas the step-by-step method only applies to first order RC or RL Circuits.

Steps to Transient Circuit Analysis:

1. Short the inductor (or capacitor in an RC circuit) and find the current in the appropriate (pertinent) part of the circuit through node or loop analysis at t =0- (i.e. the instant before the switch is flipped)

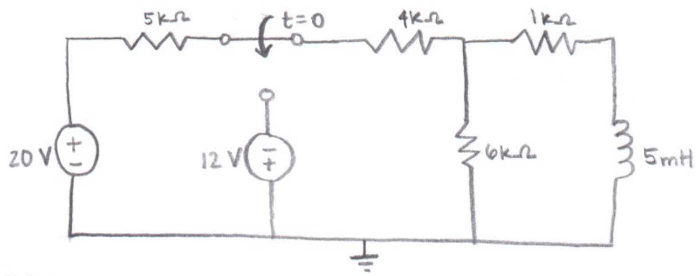
2. Again short the inductor and find the current in the appropriate parts of the circuit at t = ∞ (i.e. a long time after the switch has been flipped a.k.a. steady state)

3. Derive a differential equation from the furthest simplified $t = 0^+$ circuit (will be denoted as t > 0 hereafter); the instant after the switch has been flipped, to describe the behavior of the current or voltage in the circuit. The general equations are:

   → $V(t) = LI' + RI + \frac{x}{C}$, where L is the nominal value of the inductor in henrys, C is the value of the capacitor in Farads, x is the charge on the capacitor (x' = I, current in Amperes), and V(t) is the function of an AC voltage source, or a constant number (e.g. 20V) if a DC source. V(t) = 0V results in a homogeneous Ordinary Differential Equation (ODE) if the circuit lacks a voltage source. If the Circuit lacks any component (is only first order as in this case), such as an inductor or resistor or capacitor, that corresponding value is zero in the general V(t) function.

   → $V'(t) = I(t) = LI'' + RI' + \frac{I}{C}$ This equation is the derivative of the V(t) function above.

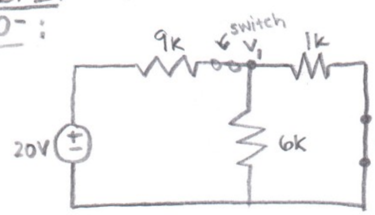
   Depending on what the problem asks, either or both of these may be used to find the voltage or currents in any given circuit.



ORIGINAL CIRCUIT

STEP 1:

$t = 0^-$:



$9k = 5k + 4k$ resistors in series

← inductor is shorted

↑ second voltage source does not affect circuit when switch is in this position.

* Node Equation at V_1:

KCL @ V_1:

$$\emptyset = \frac{V_1 - 20\,V}{9k\Omega} + \frac{V_1 - \emptyset V}{6k\Omega} + \frac{V_1 - \emptyset V}{1k\Omega}$$

Finding common denominator + simplifying

$$\emptyset = 2V_1 - 40 + 3V_1 + 18V_1$$

$$40 = 23V_1$$

$$\boxed{V_1 = \frac{40}{23}\,V}$$

(V) The 1kΩ resistor is used here because the goal is to find the current that flows to the inductor; in the absence of the inductor, the current where the 1kΩ resistor would be equilvalent to the current that would pass through the inductor.

* Using Ohm's Law: $V = IR$

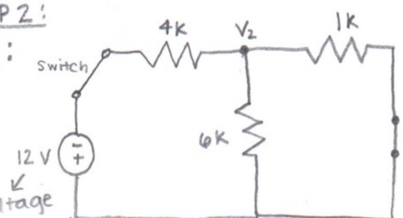
$$I = \frac{V}{R} = \frac{\frac{40}{23}V}{1k\Omega} \text{ (V)}$$

$$\boxed{I_{0^-} = \frac{40}{23}\,mA}$$

≈ initial condition
↳ initial current through inductor

STEP 2:

$t = \infty$:



← inductor is shorted

only voltage source involved when switch is in this position.

* Node Equation (KCL) at V_2:

$$\emptyset = \frac{V_2 + 12V}{4k\Omega} + \frac{V_2 - \emptyset V}{6k\Omega} + \frac{V_2 - \emptyset V}{1k\Omega}$$

common denominator + simplify

$$\emptyset = 6V_2 + 72 + 4V_2 + 24V_2$$

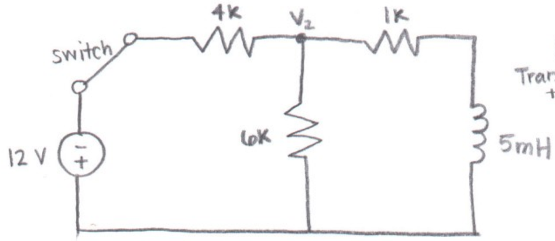
$$-72 = 34V_2$$

$$V_2 = \frac{-72}{34} = \boxed{\frac{-36}{17}\,V}$$

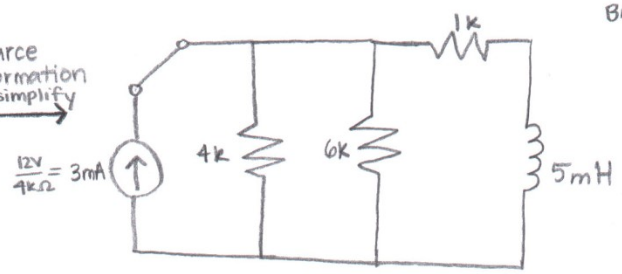
* Using Ohm's Law: $V = IR$

$$I = \frac{V}{R} = \frac{\frac{-36}{17}V}{1k\Omega} = \boxed{I_\infty = \frac{-36}{17}\,mA}$$

$t>0$:



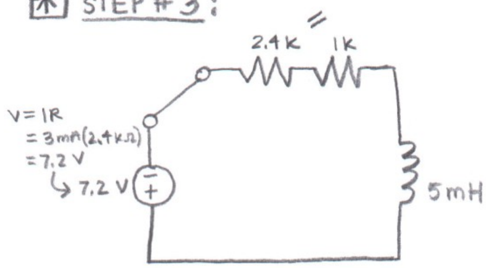
Source Transformation to simplify →



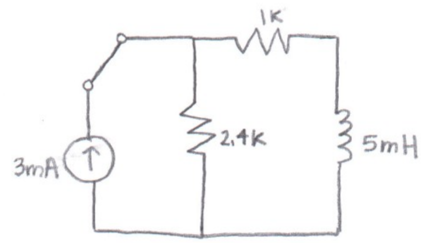
↓ simplification: resistors in parallel

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_n}$$

✱ STEP # 3:



← Source Transformation to further simplify



$$\frac{1}{R_T} = \frac{1}{4k} + \frac{1}{6k}$$

$$R_T = 2.4\,k\Omega$$

Derive Differential Equation from above circuit using Kirchoff's Loop Law:

Solve using Undetermined Coefficients Method

Note: since the circuit lacks a capacitor, there is no $\frac{x}{C}$ term in the differential equation

$$\emptyset = -7.2V + (3.4k\Omega)\,i + (5mH)\frac{di}{dt} \Rightarrow \text{General Homogeneous Equation for R-L Circuit}$$

→ characteristic equation:

$$\emptyset = 3.4 \times 10^3 + 5r$$

$$r = -6.8 \times 10^5$$

⇒ often in circuits analysis the differential equation is written as:

$$V(t) = k_2 + k_1 e^{\left(-\frac{t}{\tau}\right)} \quad \text{where } \tau = RC \text{ or } \frac{L}{R} \text{ (time constant)}$$

$$\tau = -\frac{1}{r} \leftarrow \text{root from characteristic equation}$$

$$I_H \overset{\text{homogeneous}}{=} c_1 e^{-6.8\times10^5 t}$$

particular → $I_p = A \leftarrow$ some constant

$$I'_p = \emptyset$$

$$I(t) = \overbrace{\frac{-36}{17}}^{I_\infty} + c_1 e^{-6.8\times10^5 t}$$

$$I(\emptyset) = \frac{-36}{17} + c_1 e^0 = \left.\frac{40}{23}\right\} I_{0-} \leftarrow \text{"initial condition"}$$

$$c_1 = 3.86$$

$$\boxed{I(t) = \frac{-36}{17} + 3.86 e^{-6.8\times10^5 t}} \Rightarrow \text{particular solution}$$