Calculus in Golf

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Calculus can determine the motion of a golf ball given a few initial factors. Using only launch angle and ball speed, we can determine the distance the ball will travel, the height it will reach, and the time it will take. By breaking the motion of the ball into horizontal and vertical components, we can easily determine the ball's position at any one time.

The acceleration of any object undergoing simple projectile motion is the acceleration due to gravity, which scientists have determined is -9.8 meters per second squared and acts only in the vertical direction. The horizontal acceleration of the ball, neglecting air resistance and spin, is zero.

$A_x(t) = 0$
$A_y(t) = -9.8$

Taking the integral, or antiderivative, of the acceleration gives us the instantaneous velocity of the ball.

$V_x(t) = v_{xo}$
$V_y(t) = v_{y0} - 9.8t$

Given the initial velocity, we can determine the instantaneous velocity of the ball at any time t. The velocity equation can be integrated to get the position equation, which can be used to determine the position at any time t.

$X_x(t) = x_{x0} + v_{x0}t$
$X_y(t) = x_{y0} + v_{y0}t - 4.9t^2$

The velocity of the ball, combined with the launch angle, can be broken into horizontal component v_x and vertical component v_y. Using simple trigonometry, we can figure that the initial horizontal velocity, v_{x0} is equal to the initial velocity times the cosine of the launch angle. The vertical velocity v_{y0} is equal to the initial velocity times the sine of the launch angle. To further simplify the equations we shall set the origin equal to the initial position of the ball and set time t=0 as the moment the ball is struck. As a result we can deduce that x_{x0} and x_{y0} are both equal to zero. Using this information we can get the equations for $S_x(t)$ and $S_y(t)$:

$S_x(t) = v_0\cos(\theta)t$
$S_y(t) = v_0\sin(\theta)t - 4.9t^2$

And in Cartesian coordinates, the position of the ball at any given time t is:

$(v_0\cos(\theta)t, v_0\sin(\theta)t - 4.9t^2)$

Using the equations derived above, we can figure out the time it will take the ball to hit the ground by establishing when the vertical velocity is equal to the opposite of the initial vertical velocity, since the ball follows a parabolic curve. If we set $v_y(t)$ equal to zero and solve for t, we will have the time at which the ball hits the ground. Using $x_x(t)$ and plugging in that time t, we can establish the distance the ball travels in the air. To find the maximum height of the ball, we can find the time t when the vertical velocity of the ball is equal to zero and plug that value of t into $x_y(t)$. Interestingly, since the ball traces a parabola, the apex of its flight will occur at a time equal to one half the hang time.

Let us use the equations we derived plugging in numbers from professional golfers to determine how accurate they are. The average ball speed is 164 miles per hour, which is 73 meters per second. The average launch angle is 11.2 degrees.

Plugging in 73 meters per second for v_0 and 11.2 degrees for θ, we learn that theoretically the ball should travel 207.2 meters in 2.9 seconds and it should reach an apex of 10.4 meters in 1.4 seconds.

The average carry distance is 268 yards, which is 245 meters. The average hang time for a pro drive is 6.3 seconds. The average apex is 89 feet, or 27 meters. The error in these numbers is a result of varied terrain (the ball may be launched from a different height from the landing zone), air resistance, but mostly ball spin (pro golfers generated anywhere from 1500rpm to 4000rpm of backspin on their drives). The spin of the ball, coupled with the dimples on the ball itself, allows the ball to stay in the air much longer and travel much farther as a result.