

SOLUTION TO PROBLEM 5 OF CHAPTER 4

We are assuming that X and Y are random variables such that $P(|X - Y| \leq M) = 1$, for some constant M , and that Y has finite expectation.

First we will show that X has finite expectation. To this end, note that Theorem 3 on page 88 applied to $X - Y$ implies that

- (1) $X - Y$ has finite expectation,
- (2) $|E(X - Y)| \leq M$.

Using (1) and the fact that Y has finite expectation, we can apply Theorem 2 (iii) on page 87 to $X - Y$ and Y to conclude that $X = (X - Y) + Y$ has finite expectation.

Next we will show that $|EX - EY| \leq M$. To do this, we apply Theorem 2 (ii) and (iii) to conclude that $E(X - Y) = EX - EY$. Using that fact and (2), we now deduce that $|EX - EY| = |E(X - Y)| \leq M$.