

#### SOLUTION TO PROBLEM 27 OF CHAPTER 4

The total number,  $X$ , of defective bolts in a shipment of 10,000 has the binomial distribution with parameters  $n = 10000$  and  $p = 0.05$ . We want to find the smallest integer  $a$  so that  $P(X > a) \leq 0.01$ . We will apply Chebychev's Inequality, (26) on page 101, to estimate  $a$ .

We have  $\mu = np = 500$  and  $\sigma^2 = np(1 - p) = 475$ . Thus

$$\begin{aligned} P(X > a) &\leq P(X \geq a) = P(X - 500 \geq a - 500) \\ &\leq P(|X - 500| \geq a - 500) \leq \frac{475}{(a - 500)^2}. \end{aligned}$$

Solving the inequality  $475/(a - 500)^2 \leq 0.01$  for the smallest integer  $a$ , we find that  $a = 718$ . *Note:* In Exercise 46 of Chapter 7, we will use the Central Limit Theorem to get a significantly better estimate of  $a$ .