SOLUTION TO PROBLEM 1 OF CHAPTER 1

(a) Let A be a subset of Ω containing m elements, say, $\omega_1, \omega_2, \ldots, \omega_m$. Then we have

$$A = \{\omega_1,\ \omega_2, \ldots,\ \omega_m\} = \{\omega_1\} \cup \{\omega_2\} \cup \cdots \cup \{\omega_m\},$$

and, because the events $\{\omega_1\}, \{\omega_2\}, \ldots, \{\omega_m\}$ are mutually exclusive,

$$P(A) = P(\{\omega_1\}) + P(\{\omega_2\}) + \cdots + P(\{\omega_m\}) = p + p + \cdots + p = mp.$$

On the other hand, we must have $P(A) \leq 1$. Therefore $mp \leq 1$ or $m \leq \frac{1}{p}$.
If Ω had an infinite number of points, then it would contain a subset with more than $\frac{1}{p}$ points, which we have just seen is impossible. So, Ω must be a finite set.

(b) Let $n = N(\Omega)$. Using the same type of argument as in part (a), we find that $P(\Omega) = np$. But also $P(\Omega) = 1$. So, $p = \frac{1}{n}$.